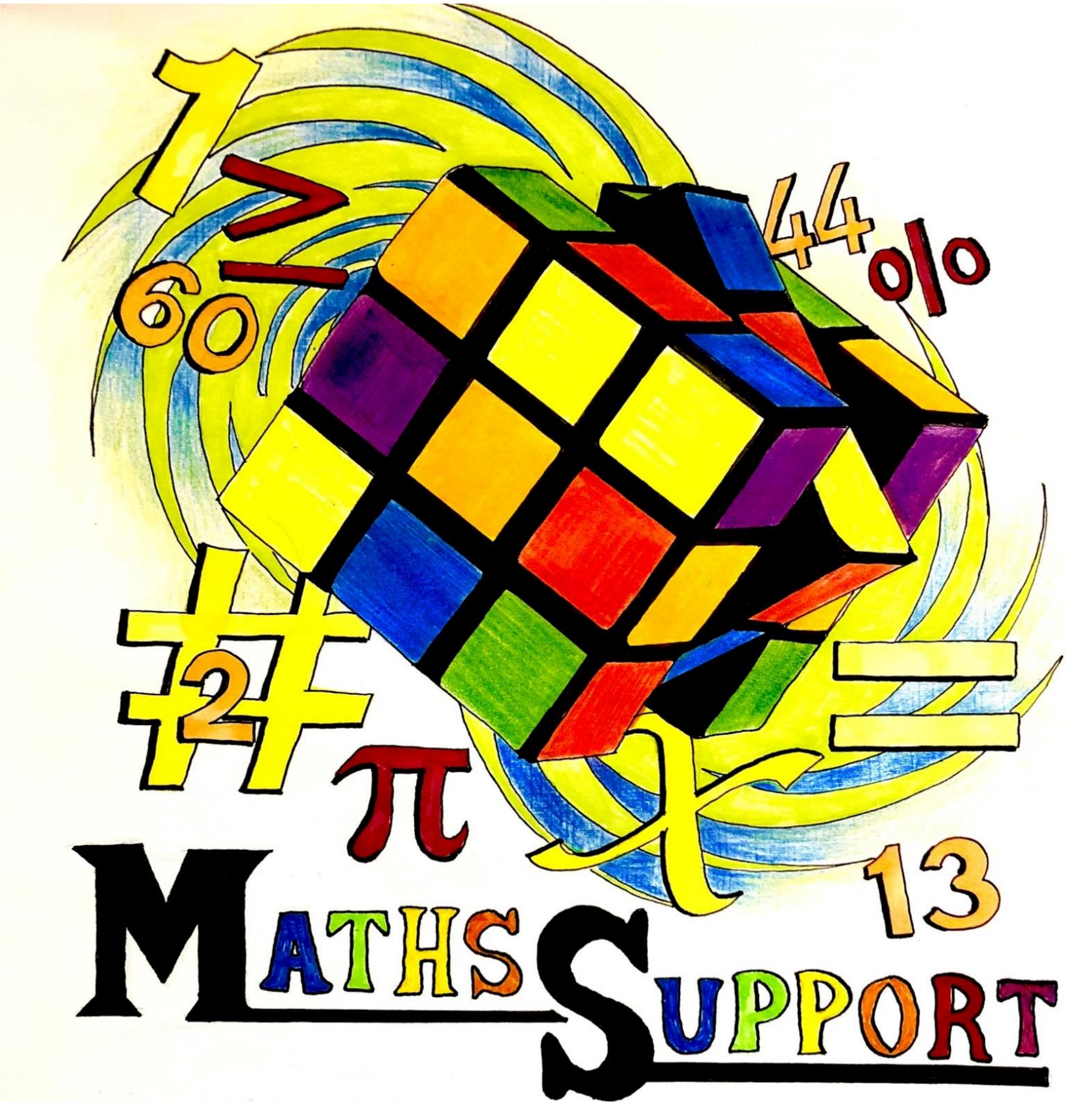


A Good Practice Guide for Teachers 2020

An Roinn Oideachais agus Scileanna
Department of Education and Skills

An tSeirbhís Náisiúnta Síceolaíochta Oideachais
National Educational Psychological Service (NEPS)

Contents

Introduction

In recent years, there have been concerns about improving numeracy standards in many countries, including Ireland. Poor maths skills are associated with high dropout rates, limited life success, increased risk of anti-social outcomes and economic costs (Commission of the European Communities, 2011).

Alongside this concern about standards, there is growing understanding of what constitutes quality maths teaching and learning. The teacher's role has changed from instructor, teaching rules and procedures and correcting related exercises, to facilitator of understanding, mathematical thinking and reasoning abilities. Maths classrooms are changing from silent, didactic, static environments to vibrant places of talking, listening, reasoning and justifying.

Various developments have impacted positively on Irish maths standards. These include curricular changes, professional development opportunities for teachers and new initiatives emphasising both understanding and real-life application of knowledge and skills. Although results in both international and national assessments (PISA 2015, NA14) have improved, Ireland's overall performance in international mathematics' studies is disappointing, especially when compared to our literacy performance. We continue to see distinct areas of weakness - most notably problem-solving.

It takes a certain energy and planning by school management to prioritise resources and interventions for students with maths challenges. *Maths Support: A Good Practice Guide for Teachers* contains advice and guidance for school staff supporting such students. It applies to all 3 to 18-year olds with different maths challenges, including, among others, those with general learning difficulties, those with maths anxiety, those with "gaps" in their knowledge, and those with Dyscalculia. It is applicable to students from disadvantaged backgrounds, to minority groups and to students for whom English is not a first language. We propose some possible causes for maths challenges, offer teaching tips and suggest intervention approaches grounded in current research findings. We include links to resources and professional development sites and to short video clips demonstrating good practice.

We acknowledge with gratitude teachers' contributions throughout Ireland, along with helpful assistance from the SESS, NCCA, PDST, Colleges of Education and the Inspectorate. All shared their experience and expertise generously. We are grateful to Ciara de Loughry, Collette Murphy, Yvonne Mullan, Aideen Carey and Valerie Jones of the NEPS Numeracy Group for producing this resource. Thanks also to Ray Mullan for his many drawings.

Important Messages
Important messages are highlighted in yellow boxes

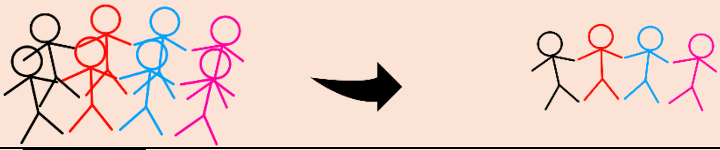
Teaching Tips
Teaching Tips are highlighted in peach boxes. If you do not want to read the more detailed information, you can move quickly from box to box to pick up teaching ideas in each section.

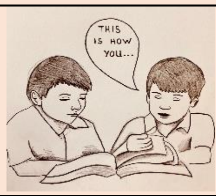
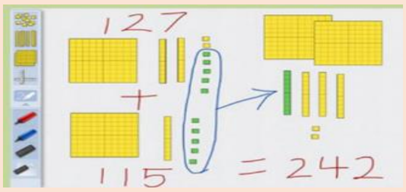
Links in the document appear in **blue font.**

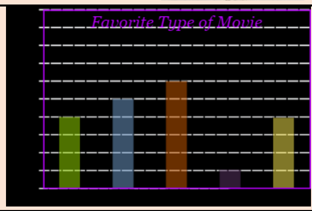
<table>
<tr><td colspan="2">

General Guidance

for students with maths challenges

</td></tr>
<tr><td>Increase teaching time for student</td><td></td></tr>
<tr><td>Reduce group/class size
Give individual support if necessary</td><td></td></tr>
<tr><td>Integrate strategies which support cognitive processes</td><td> Read more in Sections B & C of this guide</td></tr>
<tr><td>Get students to talk about decisions, strategies & solutions to maths problems</td><td>I made the numbers smaller…</td></tr>
<tr><td>Provide peer-assisted support to students</td><td>


</td></tr>
<tr><td>Use a multisensory teaching approach</td><td>  </td></tr>
<tr><td>Teach students to represent information visually in a maths problem</td><td>


</td></tr>
<tr><td>Struggling students need short-cuts & tricks as memory aids</td><td>6x7 -6 & 7 are sweatin'
on a bicycle made four two (42)

</td></tr>
<tr><td>Be systematic & explicit</td><td><u>Systematic</u> means gradually building on proficiency by introducing concepts in a logical order & by providing students with many applications of each concept.
<u>Explicit</u> means providing clear models, an array of examples & much practice when using new strategies & skills.</td></tr>
<tr><td colspan="2">Adapted from Jayanthi, M., Gersten, R., Baker, S. (2008). Mathematics instruction for students with learning disabilities or difficulty learning mathematics: A guide for teachers. Portsmouth, NH: RMC Research Corporation, Centre on Instruction.</td></tr>
</table>

Quick Guide to Challenges and Teaching Tips

Challenges	Teaching Tips
Anxiety **Feeling apprehensive and tense** **Rarely volunteering answers** **Seeking regular reassurance** **Reluctance to start** **Making wild guesses** **Avoiding maths**	Choose assessment methods & tests carefully Avoid timed tests until confidence grows Encourage maths talk and listening to others Respect errors Use process-oriented teaching (less reliance on facts & memorisation) Ensure consistent maths class routine Intervene early Be aware of teacher anxiety Revise basic facts regularly using creative methods Use games & technology Be positive & give plenty of positive feedback **Read Anxiety Section A**
Number Sense **Limited intuitive understanding of number** **Relies on recall of facts and procedures rather than on understanding the underlying concepts** **Limited flexibility with number** **Difficulty recalling basic number facts and formulae** **Struggles with estimation** **Procedural Errors** **Inconsistent calculations** **Close-to-Correct answer** **Misreading signs ÷ × + - ≤ ≥ …**	Give meaningful practice with motivating materials Do mental maths regularly Use aids until student is fluent Reduce emphasis on speed Provide small doses - daily 10-minute sessions Encourage self-monitoring & listening to peers Teach commutative law or “turn arounds” (4+5 =5+4) Teach thinking strategies from one fact to another 5+5 then 5+ 6; 3+3 then 3+4 Use number sense software Encourage students to keep track of how many and which facts are mastered Stress the *why* of procedural maths as well as the *how* Identify error type or pattern Develop self-monitoring & self-checking systems Use acronyms & mnemonics Listen to students as they “think aloud” **Read Number Sense Section B**
Reasoning **Making sense of a problem** **Knowing where to begin** **Translating a word problem into a maths “sum”** **Figuring out what to do** **Seeing that an answer does or does not make sense**	Link word problems to student interests When presenting a problem move from real-life to abstract Use concrete materials & hands-on learning approaches Get students to visualise & draw maths problems Leave space beside a problem to draw a picture of it Students create word problems (from number facts) **Read Mathematical Reasoning Section B**
Memory **Remembering birthdays** **Remembering the page to open** **Remembering number facts, procedures and formulae** **Remembering steps of a problem**	Build number fact fluency using motivating approaches Use regular reminders to help listening Use checklists notebooks & to-do lists to help stay on task Use visual and other sensory aids Teach memory aids e.g. mnemonics, rhymes, jingles Practise, revise, re-learn & rehearse **Read Memory Section C**

Quick Guide to Challenges and Teaching Tips

Challenges	Teaching Tips
Language **Following verbal instructions** **Understanding language in word problems** **Reading/ Understanding what is read** **Learning specialised terms** **Communicating their reasoning** **Communicating their difficulties/ confusion**	Slow down & be clear & concise Write instructions on the board & leave them there Read the word problems or instructions for the student Replace words with simple images or simple vocabulary Model thinking aloud when problem-solving Chunk verbal information Use visual planners (diagrams/mind-maps) Encourage use of a Maths Dictionary **Read Language Section B**
Visual Processing **Mentally rotating pictures** **Copying accurately** **Reading signs** **Identifying right & left** **Sequencing** **Reading graphs, diagrams & charts** **Filtering out background information**	Give precise & clear verbal descriptions Include multi-sensory experiences Use verbal clues for verbally strong students Use boxes, circles & lines to break up visual information Use squared paper or unlined paper depending on student Minimise or eliminate need to copy text from board Colour code written descriptions of steps in maths problems **Read Sensory Processing Section C**
Auditory Processing **Listens well but has difficulty following instructions despite adequate receptive language skills** **Finds it difficult to filter out background noise**	Reduce background noise at important listening times Check to see if student has understood instructions Use visual clues for visually strong students Provide written information on a page or board to supplement or consolidate verbal instructions Use listening devices for students with severe auditory processing difficulties **Read Sensory Processing Section C**
Executive Functioning skills **Getting started** **Staying engaged** **Remembering recent information & numbers** **Thinking flexibly** **Self-monitoring** **Getting organised** **Keeping on-track when attempting multi-step problem**	Encourage students to write down numbers in mental maths Use student check-lists & to-do lists Break long tasks into short quick sections Use memory cards & method cards Use highlighters & underlining Present problems & solutions in a variety of ways Teach mnemonics, rhymes, jingles Remind students regularly to listen, work or keep going Encourage the use of visual aids Encourage verbalisation & rechecking Teach students to question their solution **Read Executive Functioning Skills Section C**

Section A

Maths Anxiety

- **What is Maths Anxiety?**
- **Anxiety and Attainments**
- **Culture of Confidence**
- **Teaching Tips**

What is Maths Anxiety?

Basic maths skills are a necessity for success in school and in everyday life, yet many people experience apprehension and fear when dealing with numbers and mathematical information. Most of us have met people who say they are *no good at maths* or have *always failed maths in school* and who avoid maths-related tasks. An understanding of maths anxiety may help you to support a student who struggles.

Maths anxiety is a feeling of tension, apprehension and/or fear that interferes with maths performance. It can present at any stage from early years to adulthood (Ashcraft 2002). The higher a student's maths anxiety, the lower their maths learning, mastery and motivation.

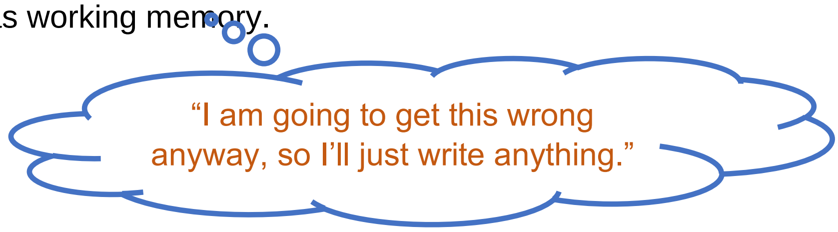
Fig.1 Maths Anxiety

It is understandable that students who are not competent in maths are likely to be more anxious about maths. However, maths anxiety can come from other sources too, such as home, school and classroom environments, where students pick up cues from parents, teachers or peers that maths is stressful (Lyons & Beilock, 2012). When parents are anxious about their own maths ability, they may pass on their own fears subconsciously. They may not consider talking to children about the number of grams in a kilogram when baking, or of asking questions such as "how many socks in three pairs?" Some teachers have maths anxiety too and inadvertently pass it on to their students (from pre-school onwards) through comments, behaviours and teaching practices (Geist, 2015). Maths-anxious teachers can have lower achievement expectations for their students (Martinez, Martinez and Mizala, 2015). They can often stick to traditional and rigid forms of teaching, overemphasising rote-learning and spending less time attending to students' questions (Bush, 1989).

Girls tend to lack the self-confidence in science and maths displayed by boys (OECD, 2015). Even from a fairly young age, girls tend to be less confident and more anxious about maths. Moreover, these differences in confidence and anxiety are larger than actual gender differences in maths achievement (Ganley & Lubienski, 2016).

Anxiety and Attainments

Maths anxiety and maths achievement are related. Anxiety can affect maths performance by impacting on student motivation, avoidance and/or executive functioning skills such as working memory.



Avoidance

Students with maths anxiety tend to avoid situations involving maths. These students may appear to daydream as they shut down during maths class. They may complain of headaches or request frequent bathroom breaks. This avoidance leads to less exposure to teaching and to practice, less competency in maths tasks and poorer maths test performance.

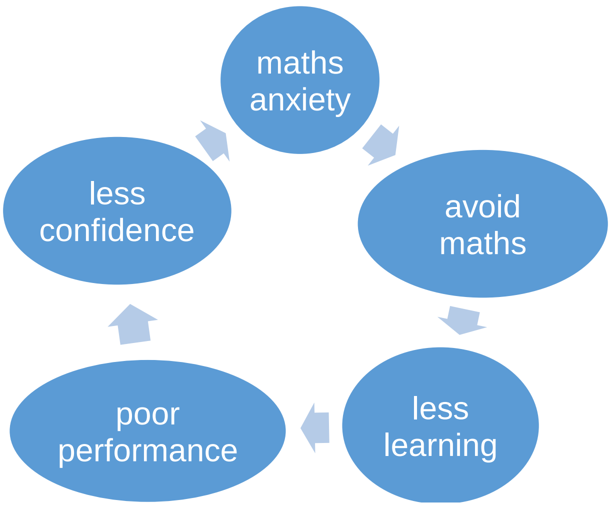


Fig. 2 Anxiety Cycle

Working Memory and other Executive Functions

Fig. 3 Anxiety Drain

Anxiety interferes with maths performance because it robs people of working memory (Beilock, 2014). Working memory is like a store that keeps several bits of information in mind simultaneously, so that a person can manipulate information to solve problems. Valuable working memory space may be taken up by anxious thoughts about failing, about not having enough time, or about what peers may think. These thoughts may diminish working memory stores available to devote to the maths problem (Beilock and Willingham, 2014). Maths anxiety also impacts on other executive functioning skills such as starting work, organising yourself, attending, engaging, prioritising and thinking flexibly. It is hard to think flexibly when you are anxious, difficult to generate new ideas when you are stressed and actively demotivating to try to engage in a task when you believe that your engagement is futile.

Create a Classroom Culture of Confidence

Students' feelings about themselves and about their learning experiences may influence their efforts and their success levels. The best confidence-building mathematics environment is one in which:

- Ω Mistakes are allowed. Students feel safe enough to suggest incorrect answers, knowing that teachers value their ideas, efforts and processes.
- Ω Academic, social and emotional skills are all fostered
- Ω Adult-student and peer relationships are kind, caring and respectful
- Ω Responsive adult feedback supports and extends student learning
- Ω Students obtain support through peer and adult discussion
- Ω Teacher-directed and student-directed activities are balanced
- Ω Teachers provide sufficient explicit mathematical experiences for struggling students
- Ω Teachers do not project their own anxieties about maths (Geist 2015).

Adapted from NCCA 2016

Teaching Tips	Maths Anxiety

- Ω Choose assessment strategies carefully (See Assessment Chapter).
- Ω Tests, especially timed tests, are one of the main causes of maths anxiety.
- Ω Identify at-risk students early on and use targeted interventions to prevent maths anxiety from developing or escalating.
- Ω Interventions may simply be *more time* or *more support* from a teacher.
- Ω Provide consistent classroom routines in maths class.
- Ω Teach *within* a student's zone of proximal development (See below).
- Ω Road maps can help to lessen anxiety as students work through problems in a step-by-step manner.
- Ω Pair students with allies who are kind and willing to help.
- Ω Be aware that teachers' feelings about maths can convey indirect messages to students through teaching methods.
- Ω Professional development and peer support can improve teacher skills and confidence, leading to more classroom enthusiasm and increased positivity.
- Ω Respect errors and sound reasoning.
- Ω Parents encourage children's basic maths skills through counting, weighing, measuring and sharing. Click **here** for a handout for parents.
- Ω Ensure that students know the basics before progressing to the next level.
- Ω Revise basics regularly with struggling students.
- Ω Link maths to real-life situations**.** Encourage estimation of price totals when shopping or measuring skills during baking and woodwork.
- Ω See **Mathseyes** for inspiration on how to make maths become real and meaningful.
- Ω Encourage a belief (mindset) that talents and abilities are not fixed, but can be developed. Read more **here.**
- Ω Games can help students forget that they're actually using maths strategies. Games such as Yahtzee, Battleship, Dominoes and Connect Four demand simple mental maths and problem-solving skills.
- Ω Be cautious about using fast-paced number or spatial reasoning games.
- Ω Remember to differentiate and choose games carefully for weaker students.
- Ω Click **here** for a list of numeracy apps from the University of Edinburgh or **here** for a list from UrAbility.

Zone of Proximal Development (Vygotsky,1978)

The zone of proximal development is the distance between the actual level and the potential developmental level of a student. It is the difference between what a student can do without help and what they can achieve with guidance and encouragement from a more skilled person. The "teacher" needs an understanding of what the student can achieve alone as well as what they might achieve with help. Then, through guidance, activities, interaction and questions, the student moves from being unable to do a task to being able to do it.

Assessment

- **What Needs to be Assessed?**
- **Purposes of Assessment**
- **Methods of Assessment**

Assessment is more than the task or method used to collect data about students. It includes the process of drawing inferences from the data collected and acting on those judgements in effective ways (Callingham, 2010). Teachers need to collect, document, reflect on and use evidence of students' learning to inform their work and provide appropriate learning experiences to ensure student progress.

What needs to be assessed?

A mixture of problems and challenges may lie beneath a student's mathematical challenges. Factors such as educational opportunity, school attendance, medical and physiological needs, anxiety, quality of teaching and the match between teaching style and individual learning styles need consideration. Checklists, questionnaires, parent-teacher and care-team meetings can gather much of this information. Checklists for basic needs, classroom-support and school-support can be found in **Appendix 2 on page 69**.

Fig. 4 What needs to be assessed?

Other important factors contributing to maths learning, such as memory, language, executive functioning and sensory processing are considered in **Section C**.
The frequency and types of assessment used in maths classes will be guided by student need, teacher expertise, teacher preference and by the school assessment plan.

A school assessment plan should contain details about the following:

- Ω When assessment occurs
- Ω How assessment data is recorded (digitally? on paper? portfolios?)
- Ω Assessment instruments that can be used
- Ω Investment in professional development
- Ω Aspects of maths that are assessed formally/informally
- Ω How information is shared with parents and guardians
- Ω How information is transferred (with permission) between schools

Click **here** for NCCA Primary and **here** for NCCA Junior Cycle reporting guidelines

Purposes of Assessment

When we assess, we generate data about a student. When the data generated is used to report on student learning at a particular time, for example at the end of an instructional unit or end of year, this is *Assessment of Learning* (AoL) or Summative Assessment. It gives us data on attainment. When the data generated by the assessment is used to inform teaching and learning this is *Assessment for Learning* (AfL) or Formative Assessment. Williams (2015) suggests that assessments themselves are neither formative nor summative. How assessment data is used, and the type of inferences formed, make an assessment formative or summative.

The most common purposes of assessment are to:

- Ω Identify students falling behind
- Ω Find out those needing extra support
- Ω Monitor a student's progress over time
- Ω Give feedback to parents or guardians
- Ω Measure an intervention's effectiveness
- Ω Observe and analyse student errors to inform teaching
- Ω Evaluate staff professional development needs
- Ω Know where to allocate school resources
- Ω Inform School Self-Evaluation
- Ω Encourage students' own self-evaluation
- Ω Collect information about factors which may be influencing performance.

Teacher Feedback to Students

Effective feedback from teachers is **clear** and **precise**.
It communicates (either verbally or in writing) which specific aspects of a task students performed correctly/ incorrectly. This type of feedback is known as **process-directed** as opposed to **person-directed** feedback. It is more effective when given during or immediately after a task is completed.

Methods of Assessment

Fig. 5 Paper and pencil

Screening is a process used to identify individuals needing further evaluation and/or educational intervention. Screening instruments are usually easy to administer to groups and can be completed in a relatively brief time. They can include pencil-and-paper tests, rating scales and checklists, or they may involve direct observation of skills or abilities. Screening tests can be standardised or more informal teacher-designed assessments of student knowledge, skills or behaviour.

Diagnostic Assessment is used to find out what exactly a student knows, what they can and cannot do and where understanding is breaking down. Teachers can diagnose specific areas of difficulty **formally** using diagnostic tests, or **informally** using many of the methods outlined in this section. When not using a test, teachers need to comprehend the skills involved in a task. They can then analyse students' errors and misunderstandings.

Response to Intervention (RTI) is an approach that allows the teacher to monitor how a student responds to instruction and then modify their teaching accordingly. It is a form of assessment **for** learning. It can impact significantly on learning when properly employed in the classroom (Black and William, 1998a). RTI is a cycle of quality teaching *and* assessment *and* modified instruction. Its primary objective is to prevent problems by offering the most suitable teaching. Such assessment is typically done through short, quick, classroom-based assessment by the class teacher. If a student is not responding to instruction as expected, then instruction is differentiated to take account of the student's needs.

Observation provides qualitative information about types of errors and reasons for errors. Teachers observe errors and misunderstandings in oral work, written work and drawings. Observation can be done in a student's presence or in their absence, but the best insights into students' errors come from listening to students as they reason aloud. Observation can also help to determine if any factors such as anxiety or physiological needs might be influencing performance.

Recording Observations

Ω **Date** all observations Ω Post-it notes Ω Rubrics Ω Class-List with a blank column for recording observations Ω Objectives checklist Ω Dated samples of work in student files Ω Students record the teacher's feedback in journals/copies

Drawing: Some students may like to show what they know by creating a drawing or diagram to demonstrate their understanding. They can be encouraged to share their thinking about what they are drawing.

Conversing: Talking with a student and listening carefully can inform a teacher about a student's reasoning, understanding and ability. It is easy to see *what* the error is in Figure 6, but clarity as to why the error occurred will only become clear when you hear the thinking behind it. The reason could be poor number sense, a well-practised but incorrect procedure and/or an incorrect formulation of words (e.g. 3 *from* 9 instead of 3 *take away* 9 or take 9 *from* 3). The teaching *fix* for each of these possible causes may differ.

$$\begin{array}{r} 63 \\ -29 \\ \hline 46 \\ \hline \end{array}$$

Fig. 6 ***What*** **is the problem?**

Interview: This is a slightly more formal discussion with a student, where target questions are determined ahead of time, ensuring that information related to a goal or learning outcome is obtained. As with all formative assessment methods, notes are taken for later reference when planning instruction. Questioning can be open or closed. *How many degrees are there in a right angle?* is an example of a closed question. The expected answer is predetermined and specific. In contrast, open-ended questions allow more than one correct response and elicit a different kind of student thinking, e.g. *Can you think of a few different ways to find the distance from the school to the shop?*

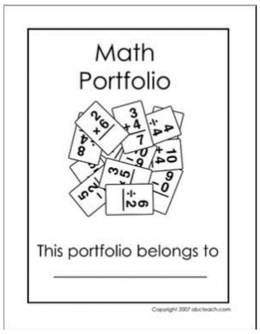


Fig. 7 Maths Portfolio

Portfolios are collections of work that show the progress made by the learner over time. The evidence may be dated samples of written work, completed teacher-composed tests, photos, video or audio records, or any other appropriate indication of the learner's achievements.

Performance Tasks are assessment tasks that require application of knowledge and skills, not just recall or recognition. They are open-ended and there is typically not one single way of doing the task. Often multiple steps are involved and several learning outcomes can be assessed. See an example of a performance task for US 2^{nd} Grade in Fig. 8.

The Bike Shop

You go to a shop that sells tricycles.
There are eighteen wheels in the shop.
How many tricycles are in the shop?
How did you figure that out?

Fig. 8 Performance Task

Find more performance tasks for various age ranges and topics **here.**

Classroom-Based Assessments (CBAs): These are performance tasks that have recently been introduced as part of the Irish Junior Cycle Programme. They require students to develop and demonstrate their knowledge and skills. An Assessment Toolkit is provided to teachers for guidance in judging student attainment. Two CBAs in Mathematics (one in 2^{nd} Year and one in 3^{rd} Year) are assessed as part of the Junior Certificate Examination.

Reflective Journals: These are useful for both teachers and students to assess thoughts, understandings, feelings and challenges. Students may need prompts to start off the writing e.g.

*Today, something new I learnt was*____

It *was easy/difficult for me*

The *tricky part was*___

What *I need to do* is___

Teachers may collect journals periodically to discern a student's performance progress in terms of their knowledge, understanding, feelings and needs.

Self-Assessment: When students self-assess, they take some responsibility for their own learning, using lists of objectives, checklists of steps or samples of completed work. They can also be taught metacognitive strategies. Such strategies help students to think about what they are doing, identify their problem-solving methods, evaluate their understanding of mathematical processes and identify *breakdowns* in their understanding. Self-assessment needs to become a routine part of what students do during and after their learning.

Peer-Assessment: Peer-Assessment is the assessment by students of one another's work with reference to specific criteria. It involves more than inserting ticks or crosses, or supplying the correct answers to each other. It is about commenting, getting ideas from others, making suggestions and asking questions which lead to revision and improvement of work. The process needs to be taught and students need opportunities to practise it regularly in a supportive and safe classroom environment. Teachers should negotiate and agree ground rules with the students. Click **here** to watch a PDST clip about peer assessment in a secondary school. You may be inspired to use something from this graphic design class in your maths class.

Section B

Number Sense

- **What is Number Sense?**
- **Build Number Sense Slowly**
- **Teaching Tips**

What is Number Sense?

Number Sense is an intuitive sense or a “feel” for numbers. Bobis (1996) describes it as a competency with numbers that is based on understanding rather than on memorisation or recall of facts and procedures. Students with good number sense are fluent and flexible with numbers. They understand the magnitude of numbers, how they relate to each other and the effects of operations on them. They spot unreasonable answers and estimate well, see connections between operations like addition and subtraction, and multiplication and division. They understand how numbers can be taken apart and put together again in different ways. For example, to add 39+51, they might quickly add one to 39, subtract 1 from 51 and add 40+50 to get the answer 90. Students who have limited number sense have trouble developing the foundations needed for simple arithmetic and for more complicated number work such as fractions and algebra (Burns, 2007). Many Dyscalculia definitions include this lack of intuitive number sense (Emerson & Babtie, 2013, Butterworth, Sashank & Laurillard, 2011). Regardless of a Dyscalculia diagnosis, if your observations and assessment indicate that a student’s difficulty is with number sense, try some of this section’s teaching tips.

Fig. 9 Number Sense

Build Number Sense Slowly

Developing number sense takes time. It begins through early experiences when children meet numbers in various contexts and relate to numbers in different ways. From concrete experiences (grouping, matching, counting, composing and decomposing) and talking about these experiences, students build foundations for computation, problem solving and reasoning. Baroody, Bajawa & Eiland (2009) suggest that we move progressively from counting to reasoning strategies and then to automaticity when working out number facts. Automaticity means that you have performed a calculation (e.g. 3+4 = 7) so often that it becomes automatic. Similar to knowing sight words when reading, knowing number facts frees up your mind to consider other aspects of maths questions. However, going too quickly from counting to automaticity can hamper reasoning strategy development. Too much speed here may result in students memorising number facts in the short-term but regressing to basic counting when they cannot recall facts in the long-term (Baroody, 2006, Henry & Brown, 2008).

The value of memorising tables (of number facts) is debatable. Some research (Ashcraft, 2002, Boaler, 2014, Ramirez et al., 2013) suggests that timed tests and emphasising fact memorisation can cause maths anxiety. Others (Stripp, 2015) believe that lack of number fact knowledge causes maths anxiety. Practice *really* does help students to recall number facts, and practice improves fluency by activating and strengthening neural networks (Aubin, Voelker and Eliasmith, 2016). We recommend avoiding bland number fact memorisation and supporting students in building reasoning and automaticity through slowing down and through making practice fun. Watch this inspirational **short video clip**.

The relationship between conceptual understanding and procedural knowledge is another hotly-debated topic. Rittle-Johnson, Schneider, & Star (2015) propose a bidirectional relationship between the two. Others (Wright, Martland, Stafford and Stranger, 2012) advocate that learning about algorithms (formal written procedures e.g. short or long multiplication or division procedures) should be delayed until students first develop their own informal strategies for combining and dividing numbers.

Using age and ability appropriate aids such as 100 or multiplication squares, number lines, along with digital aids such as calculators and maths applications can increase students' access to number facts, number sense and confidence.

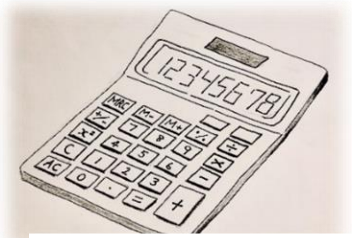


Fig.10 Calculator

Teaching Tips Number Sense

- Slow down! Some students become anxious by requests to answer quickly.
- Read PDST advice about teaching number sense **here**
- Find exactly where a student is in terms of number in **First Steps** Diagnostic Map
- Read really good practical advice for teaching students with Dyscalculia **here**
- Practise mental maths regularly; mental maths builds knowledge about numbers and numerical relationships
- *Number Talks* (classroom conversations around purposefully-crafted computation problems - Parrish 2014) are a powerful strategy for developing number sense.
- Encourage students to listen to peers when they talk about computation strategies.
- Encourage students to explain their thinking/reasoning.
- Listen carefully to students' reasoning for formative assessment purposes.
- Make estimation an integral part of computing. Real-life maths relies not only on mental maths but on estimation e.g. deciding when to leave for school, how much paint to buy, or which queue to join at the supermarket.
- Include maths facts practice *without* time pressure.
- Use technology to teach number sense. Click **here** to see First Class using iPads, **here** for UrAbility apps and **here** for software suggestions from PDST.
- Maths Recovery, Ready, Set, Go – Maths and Number Worlds are researched-based interventions for developing number sense in young children.
- Counting is important in the development of number sense. Count up, count from left to right and in a clockwise direction with young children. Count backwards too.
- Young children enjoy using a large sponge dice to decide the number of jumps to take on a large number-ladder (picture) on the floor.
- Read *Teaching Number in the Classroom with 4-8 Year Olds* by Wright, Martland, Stafford and Stranger (2015).
- Move from Concrete to Pictorial to Abstract (CPA) when developing new concepts and skills; the rate of progression from one stage to the next will vary based on the needs of individual students.

Mathematical Reasoning

- **What is Mathematical Reasoning?**
- **Representation and Reasoning**
- **Developing Reasoning**
- **Teaching Tips**

What is Mathematical Reasoning?

Reasoning means thinking about something and making sense of it in order to draw conclusions or make choices or judgements. In maths, this usually involves thinking critically about situations, words, shapes or quantities and then analysing, interpreting and evaluating. It involves using relevant prior knowledge, developing solutions and judging the solutions' accuracy. Sometimes considerable mental work is involved in the elaborations and judgements required. Creativity, imagination, memory, confidence, perseverance and ability to justify your thinking are all essential components of the process.

Is this shape a square? In this question, reasoning involves recall of prior knowledge of the term *square* and a square's characteristics. Ideally, a student has acquired this knowledge following lots of hands-on activities as a child, involving comparing and contrasting, searching for patterns, making generalizations, testing, validating and justifying conclusions.

Two friends started walking from the house. The boy walked 3k one way. The girl walked 5k the other way. How far apart were the two friends?

In this question, reasoning involves
1) understanding verbal information and then
2) figuring out how to calculate the distance between the two children. The problem's visual representation given to students in this research study helped them to work out the answer.

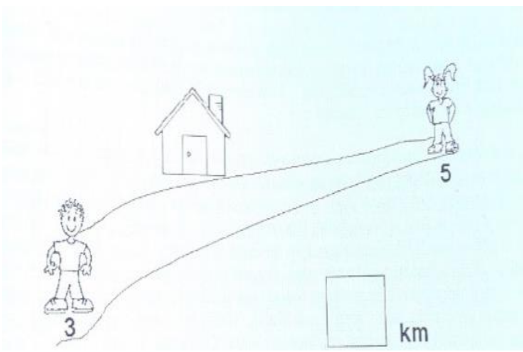


Fig. 11 An item from the Bryant 2009 study

Egg-boxes	**Minibuses**
Eggs are sold in boxes of six.	**46 children are going to a hurling match**
I have 45 eggs	**A minibus can take 12 children. How**
How many boxes can I sell?	**many minibuses will they need?**

In the questions above, reasoning again involves making sense of verbal information. The student must calculate how many boxes are needed for the eggs and how many minibuses are required for the match (quantitative reasoning). Meaningful answers will not result from merely dividing 45 by 6, and 46 by 12. Common-sense is required also. Students must remember that eggs are not sold in half-boxes and you cannot hire a fraction of a bus. Activities to support weaker students include chatting with peers and drawing pictures.

What is the value of x if $8 - 2x = 4$? This question's reasoning is more abstract than previous ones because the question contains both letters and numbers. Do *you* get the answer by trying different numbers in place of the **x**? Or do you get the answer by moving all numbers to one side of the equals (=) sign and all letters to the other side? If so, do you know why you do this? This type of reasoning becomes easier for *some* students when they have had previous practice of real-world problems.

Link between reasoning and how problems are (re)presented

The way in which a problem is represented or presented can affect a student's understanding of it. Real-life and concrete representations of problems support students to develop more abstract reasoning skills.

The waiter question on the next page comes from a Danish study in which 70% of 1st Year Algebra students worked out the answer to the informal real-world problem representation while only 42% solved the formal algebraic equation (Koedinger and Nathan, 2004). It is *not* necessary to replace formal with informal representations, but it is helpful to build on informal processes to support students' progress to formal algebraic symbol manipulation (Koedinger and Anderson, 1998).

Fig. 12 Real World Representation

Informal **Concrete** **Real-world**	**Sean gets €6 per hour as a waiter. One night he made €66 in tips and earned a total of €81.90. How many hours did Sean work?**
Abstract	**6x + 66 = 81.90** **x = ?**

The Dutch Iceberg Model in Fig. 13 was developed for teachers. Here, the iceberg's tip represents the symbol for three quarters ¾. The model's message is that informal and context-bound representations and experiences (coins, apple sections etc.) and pre-formal representations (e.g. number line) of the ¾ concept are necessary before students can fully understand the formal mathematical representation of ¾. Too much teaching invested into the iceberg's top (formal mathematics, sums) may be at the expense of developing greater insight into and understanding of concepts and skills.

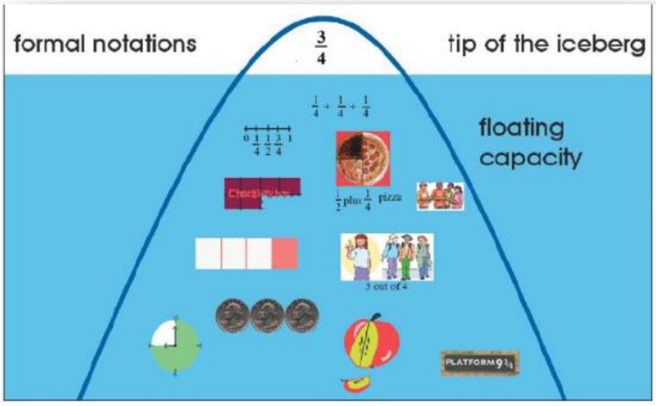


Fig. 13 Tip of the Iceberg

Developing Reasoning Skills

Ensuring that reasoning skills are developed and supported requires *active* students (Anderson, Reder & Simon, 1996). Experiences such as hands-on learning, discussion, projects and teamwork are more likely to produce lasting skills and deep understanding than passive activities such as memorisation, drill and templates.

Fig. 14 Talk and Teamwork

In Nunes, Bryant, Sylva and Barrow's (2009) study, mathematical reasoning was found to be a far stronger predictor of maths achievement than calculation skills. The researchers recommended that time be devoted to *teaching* reasoning skills in maths classes. Often in classrooms, the focus is on teaching students *how* to do maths and not on *understanding* what they do. This focus may occur because of mandated tests that emphasise calculations, pressure to ensure that students master the basics or because of teachers' perception of their own ability or expertise.

Improving student reasoning skills can reduce the anxiety often experienced in the mathematics classroom. Anxiety is lessened when individuals can control uncertainties. "When self-constructed reasoning under the control of the individual takes over, much valid mathematical reasoning may emerge" (Druckman and Bjork 1994).

Finally, an important part of reasoning is learning to communicate it in a succinct mathematical way. When students explain or justify their reasoning, they solidify their own understanding. Time is required to help students develop their language skills so that they can describe clearly their own chain of reasoning/ the sequence of steps they have taken.

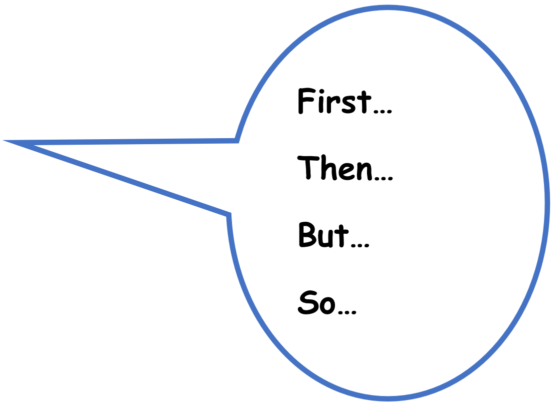


Fig. 15 Real World Maths from Mathseyes

Teaching Tips	Mathematical Reasoning

- Use real-life problems and non-routine problems to develop reasoning skills.
- Give students opportunities to talk to their peers in small groups.
- Encourage students to create problems collectively or individually.
- Click **here** for ideas from Mathseyes. This is a website from Tallaght Institute of Technology, which encourages people to develop maths eyes and spot the use of maths in everyday life.
- Include tasks which provide opportunities for analysing, evaluating, explaining, inferring, generalising, testing, validating, justifying and responding to others' arguments.
- Encourage students to use manipulative resources (post-it notes, concrete materials), pictorial representations and tables to represent the problem, investigate solutions, demonstrate understanding and justify thinking.
- Read more about representation to support student reasoning **here** and **here**.
- Find ideas to help students visualise and to draw maths problems **here**.
- Establish a classroom culture where discussion is valued and where hypotheses and conjectures can be suggested in a non-threatening way.
- Use prompts and probing questions e.g.

  What can you work out now?

  If you know that …what else do you know?

  Why is that bit important?

  If……., then……?
- Encourage students to talk to peers and teachers when they get stuck.
- Teach problem-solving strategies e.g. summarising, finding relevant data, ignoring irrelevant data, searching for clues, working backwards and trial and error ("trial and improvement").
- Model clear, succinct, logical communication of thought processes.
- Help students construct their argument by providing and displaying sentence starters:

  I think this because…

  If this is true, then…

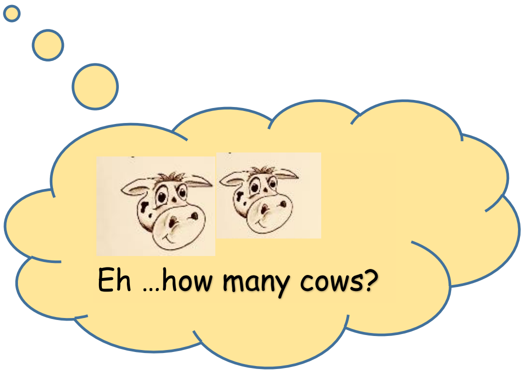
  This can't work because…

Section C

Memory

- **Long-Term Memory**
- **Short-Term Memory**
- **Working Memory**
- **Teaching Tips**

Different aspects of memory play important roles in understanding and learning maths. When you have basic number facts stored in memory, for example, it allows you to spend *less* time making simple calculations and *more* time reasoning about a problem.



One morning, there were 7 black cows in a field. In the afternoon, 2 cows left the field. Then 3 brown cows came into the field. How many cows were in the field then?

Long-Term Memory

Long-term memory refers to storage of information over an extended period. You can usually remember significant events such as a goal in a soccer game or a great concert, with much greater clarity and detail than you can recall less memorable events. Memories that you access frequently become much stronger and more easily recalled. As you access them, you strengthen the pathways where the information is encoded (Aubin, Voelker and Eliasmith, 2016). Memories that are not recalled often can weaken or be lost. It is considered that birthdays and number facts are stored in long-term memory.

Short-Term Memory

Short-term memory refers to the temporary storage of visual and auditory information for immediate retrieval or discard (Baddeley and Hitch, 1974). The amount of information that you can capture, store, process and recall in short-term memory is limited. Miller (1956) suggested that we can keep 7 ± 2 items in short-term memory.

Cowan (2005) provided evidence that a more realistic figure is 4 ± 1 items. Students with weak short-term memory may have a much lower capacity than 4 ± 1. They may experience difficulties recalling and sequencing in multi-step maths tasks.

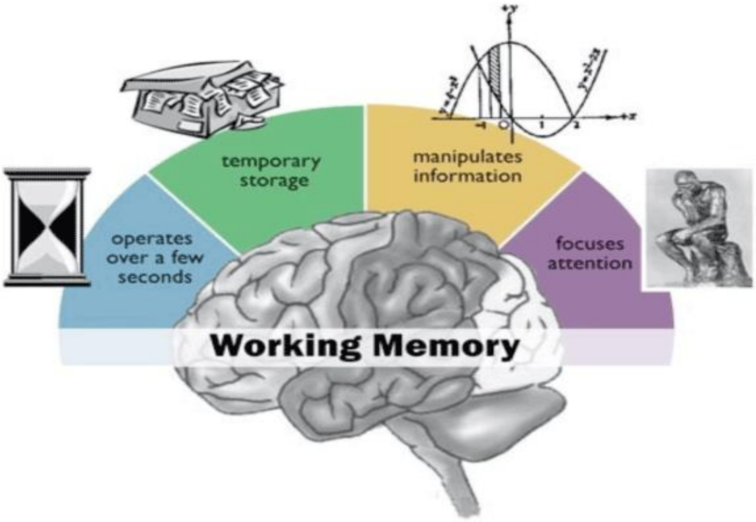


Fig. 16 Working Memory

Working Memory

Working Memory refers to the combination of storage *and manipulation* of visual and auditory information. Working memory is necessary for staying focused, blocking out distractions and completing tasks. A student's working memory in a maths class may need to deal with teacher instructions, distracting sounds and sights and/or temporary memories from long-term storage. Working memory uses information that is stored in both long-term and short-term memory.

Munro (2011) outlines teaching procedures to help students with mathematics learning difficulties to encode and manipulate their knowledge in working memory. Teachers should firstly stimulate explicitly students' current knowledge about a new task i.e. remind them of known concepts, procedures, symbols and factual knowledge they will need to use (Munro, 2011).

Processing and Storing Information

Many factors influence how we process and store information, such as:

- Ω Information complexity or volume
- Ω Information connection to long-term memories
- Ω Number of senses used in processing information
- Ω Emotional connections to the information.

Teaching Tips   Memory

- Build number fact fluency. See page 24 for suggestions and ideas.
- Reduce the load of number facts. When students understand the commutative law, for example, the number of facts to be remembered is halved 8x9=9x8. Click **here** for more ideas.
- Remember number facts in peculiar ways
  - *6 X 7 = 42 - 6 and 7 are sweatin' on a bicycle made* ***four two***
  - *8 X 8 = 64 - I* ***ate*** *and I* ***ate*** *and I got sick on the floor*
  - *Multiply by 9 - all products add up to 9 e.g. 9x2=18 (1+8=9);9x3=27 (2+7=9)*
- Allow students to use aids such as number lines, multiplication squares or calculators to check number facts or to *get* number facts.
- Cue students into listening at key points in a maths lesson.
- Avoid too much teacher talk which can lead to tuned-out students.
- Make connections between concepts and student interests/knowledge.
- Ensure students process information using as many senses as possible.
- Use visual aids from **You Cubed** (Stanford University)
- Encourage students to visualise and/or draw the problems they are trying to solve.
- Allow students to jot down numbers during mental maths.
- Revise concepts frequently.
- Break large amounts of information into smaller chunks.
- Teach memory aids such as rehearsal and mnemonics e.g. acronyms, acrostics and associations. Read more **here.**
- Encourage active reading strategies by using post-it notes and highlighter pens. Paraphrase relevant information.

Fig. 17   BIMDAS

- Repeat explanations for some students.
- Leave problems *and* ideas for solving problems on the board during a lesson. Fig. 18 shows a maths problem and 5 student-suggested ways of solving the problem. These solutions were left on the whiteboard throughout the lesson. Read more about Secondary School Lesson Study **here.**

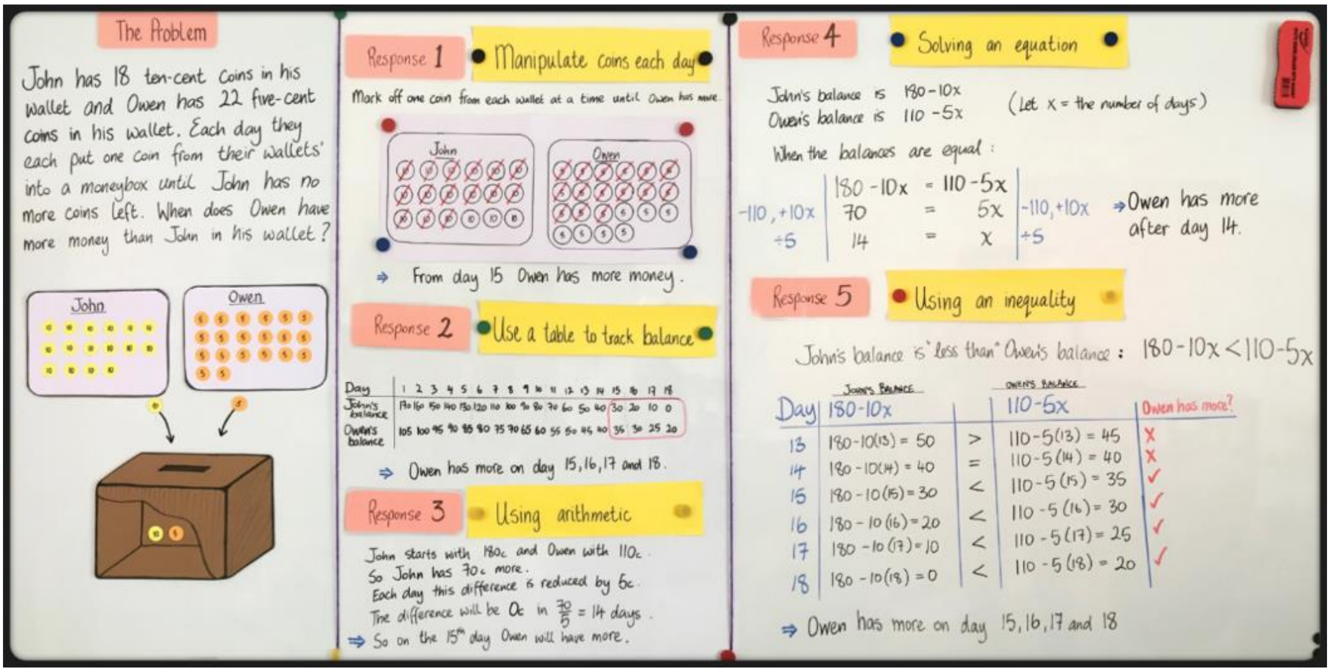
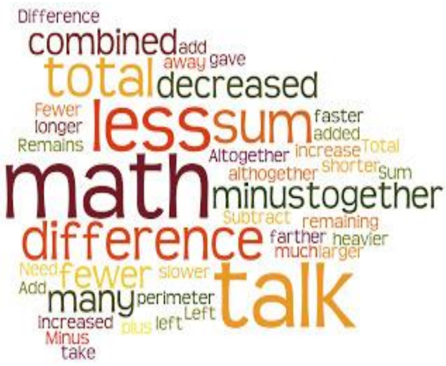


Fig. 18 Whiteboard work from Lesson Study PDST

Language

- **Vocabulary**
- **Verbal Reasoning**
- **Reading Skills**
- **Making Sense of Symbols**



Language plays an important role in mathematics learning (Schleppegrell, 2010). Language difficulties can affect a student's ability to:

- Ω Understand and make use of instruction
- Ω Solve maths word problems
- Ω Decode and interpret mathematical information
- Ω Encode and represent mathematical information
- Ω Memorise information such as number facts/terms
- Ω Reflect on their difficulties
- Ω Ask effectively for help. (Dowker, 2009)

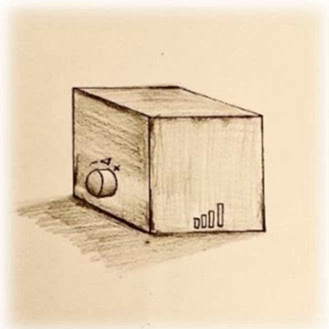


Fig. 19 The volume of a box

Vocabulary

Students may be good at computation but their ability to apply and demonstrate their skills will suffer if they do not understand the maths vocabulary used in instructions and in story problems *e.g. "How much less?" or "How much altogether*?" (Bruun, Diaz, and Dykes, 2015). Understanding words affects the understanding of concepts, so students with limited vocabulary are significantly disadvantaged. Terms such as hypotenuse, perimeter and symmetry (and their meanings) can be difficult to remember. Homonyms and homophones can confuse. Homonyms are words having more than one meaning, although they are spelt and sound similarly e.g. volume or product. Homophones are words sounding the same in speech but are spelt differently and have different meanings e.g. root, route. While some words will be learned through incidental exposure, many need explicit teaching. Words are best learned through repeated exposure in multiple oral and written contexts.

Teaching Tips	Vocabulary

- Ω Repeat information for students who need extra time to process verbal input.
- Ω Exposure to new words is most effective over an extended period of time.
- Ω A student may need as many as 17 explicit exposures to use a word comfortably (Ausubel & Youssef, 1965).
- Ω Break multi-step instructions into two or three short steps.
- Ω Use informal words or definitions alongside formal vocabulary. e.g. **Volume -** how much space does this box take up?
- Ω Link new words to prior knowledge to anchor them in stored concepts.
- Ω Introduce a new concept/word with visuals (See Fig. 20).
- Ω Encourage students to use visual planners, diagrams, summaries and mind maps.
- Ω Click **here** for JCSP's useful graphic organiser.
- Ω Use humour as a tool to anchor words and concepts.
- Ω Encourage students to record hints to remember new words in personal journals or glossaries.

Fig. 20 Hypotenuse

- Ω Encourage students to read accurately and attend to meaning e.g. of/off: 10% *of* a price & 10% *off* a price**.**
- Ω Words *sometimes* give clues about which procedure to be applied in a problem e.g. addition is associated with words such as *and***,** *altogether* and *more* but not always e.g. *How much more did you pay than I paid?* requires the use of subtraction.
- Ω *Allow students lots of talking time together.* A peer's explanation of a new word or concept can often impact on understanding more effectively than one from a teacher.
- Ω Click here for great lists of mathematical words and phrases.

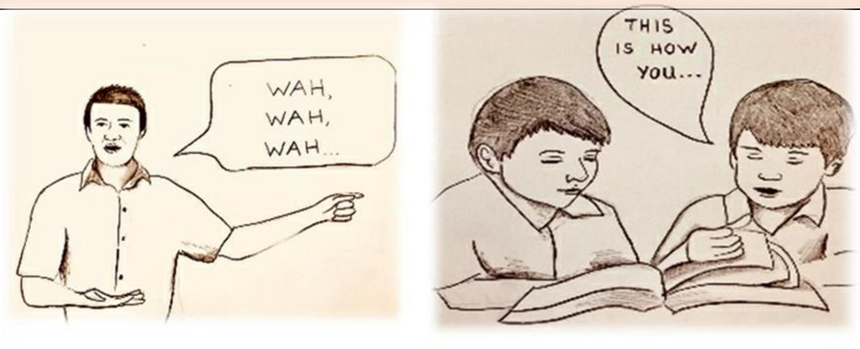


Fig. 21 Peer Talk

Verbal Reasoning

The total of the combined ages of Niamh, Ahmed and Pierre is 80.

What was the sum of their ages 3 years ago?

Was it a) 71, b) 72, c) 74, or d) 77?

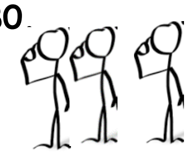


Fig. 22 Draw the problem

When you try to solve the problem above, you see that it requires more than numerical ability. First you must make sense of the words that you read or hear, and then you have to think about how to find a solution. Word problems in maths often require a student to use verbal reasoning alongside spatial or quantitative reasoning. Many will find this problem easier to work out if they draw a picture of it. Read more about mathematical reasoning in **Section B.**

Teaching Tips	Verbal Reasoning
Ω Replace unfamiliar words and topics with familiar ones e.g. "what is the area of "*the GAA pitch"* instead of "*the baseball pitch*".	
Ω Use visuals, especially pictures, drawings and diagrams.	
Ω Replace large numbers with smaller numbers to help work out what you need to do.	
Ω Encourage students to compose their own word problems, maths comics and stories as a strategy for understanding how to use words and numbers together to pose questions.	
Ω Click **here** to read a summary of George Polya's 1945 suggestions (still relevant) for helping students to reason about mathematics problems.	

Reading Skills

Weak reading skills can be an obstacle in mathematics, particularly with text-based problems. To check if reading skills are a difficulty, observe how well a student solves a problem when it is read aloud or when working with a peer who reads well. Then compare the student's performance when working independently on problems, without the reader. Click **here** to read about literacy and learning in maths.

Teaching Tips Reading Skills

- Use pair work and group work to ensure that students with weak reading skills can access word problems in class.
- Try to ascertain if a student's reading difficulties are caused by the technical aspects of reading or by language comprehension challenges, and intervene appropriately.
- Ensure that weak readers get *individualised & expert* support for reading. Include maths texts and vocabulary as part of their reading material.
- Read more **here.**

Making Sense of Symbols

Symbols are part of maths language that save time and space. Symbols are easily recognisable by students of all languages. They make maths simpler because they have only one meaning. Think about the subtraction symbol (–). There is only one way to write the symbol, but there are many words to describe the operation (subtract, minus, take away, decreased by). To help students recall the meaning of symbols, try to find ways of linking new symbols to student background knowledge.

Teaching Tips Symbols

- Find a list of symbols **here** with dates of origin and information about their origins.
- Ask students to generate their own personalised ways of recalling a symbol.
- Sigma $\sum$ is the equivalent to our letter S and means the "**S**um of".
- $\exists$ reminds us of the letter E and means *There **E**xists.*
- Infinity ∞ looks like the numeral **8** having a rest on its side because it has a never-ending journey.
- Help students recall the *less than* sign **<** showing the smaller end of the sign always points to the smaller amount or number.

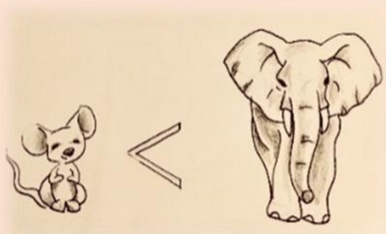


Fig. 23 Less Than

Sensory Processing

- **Auditory Processing**
- **Visual Processing**
- **Kinaesthetic and Hands-On Learning**

We learn about the world through our senses (sight, sound, touch, smell, taste, body position, movement and internal body signals). Sensory processing is a term used to describe the way in which our brains receive, organise and respond to sensory input. Many sensory processing skills play an important role in mathematical development. For example, *visual* processing skills have been shown to be an important predictor of mathematical competence (Uttal, Meadow, Tipton, Hand, Alden and Warren 2013) and some students have *auditory* processing challenges that impact on their maths learning (Bley and Thornton, 2001).

Auditory Processing

When individuals have auditory processing challenges, all the parts of the hearing pathway are working well (i.e. there is no physical difficulty) but something delays or scrambles the way the brain recognises and processes sounds, especially speech. Researchers don't fully understand where things take a different course, but language tends to be muddled and/ or normal rate of speech is too fast for the brain to process. The result is that auditory messages are incomplete or jumbled.

Fig. 25 Auditory Processing

A student with auditory processing challenges might have difficulty with

Auditory Discrimination: The ability to notice and recognise the subtle differences between similar-sounding speech sounds e.g. 70 and 17, 3 and free.

Auditory Figure-Ground Discrimination: The ability to pick out and focus on important sounds in the midst of background noise such as teacher or student voices in a busy classroom.

Auditory Memory: The ability to recall what you've heard, either immediately or when it is needed later e.g. follow verbal directions or remember maths facts.

Auditory Sequencing: The ability to remember the order of items heard. A student might hear 259 but might say or write 925, or might have difficulties remembering the correct order of a series of verbal instructions.

Auditory Attention: The ability to stay focussed on listening. The student gets exhausted with the effort exerted in trying to process what is heard.

Teaching Tips Auditory Processing
Ω Reduce background noise at important listening times. Ω Add rugs to an echoing room. Ω Use classroom visuals (pictures/ images/ gestures/ written cues/ copies of classroom notes) to aid understanding and memory. Ω Speak clearly, slow down the rate, use simple expressive sentences, maybe repeat. Ω Teach in small chunks- too much information is overwhelming. Ω Check that students have understood instructions. Ω Give the student more time to process auditory information and to complete classroom work. Ω Re-teach concepts and skills, especially multi-step processes such as long multiplication or division. Encourage regular practice. Ω Incorporate specific activities e.g. auditory discrimination or auditory memory games, to help boost auditory processing skills. Ω Encourage the student to take ownership - to ask for assistance and to self-advocate. Students need to clarify that the information has been heard correctly, ask the teacher to repeat, write notes, notice a noisy environment and move to a quieter place, look at the speaker, give friendly reminders to busy teachers. Ω Consider using computer software such as Fast ForWord for working on sound discrimination, auditory memory and language processing. Ω Use assistive listening devices (such as headphones with a wireless amplification system) for students with severe auditory processing difficulties. Ω Sometimes speech and language therapy is accessed for help to develop a student's listening skills and ability to identify sounds.

Visual Processing

Fig. 26 Visual Processing

Visual Processing, like auditory processing, is a complex function undertaken by the brain. It refers to the brain's ability to make sense of what the eyes see. This is not the same as visual acuity which refers to how clearly a person sees. Sometimes issues occur when the brain has trouble accurately receiving or interpreting visual information.   There can be a number of different issues and a student with visual processing challenges might have difficulty with

Visual Discrimination: the ability to attend to and identify a figure's distinguishing features e.g. recognise a '6' as opposed to a '9' or distinguish between coins

Visual Figure-Ground Discrimination: The ability to focus on important visual information and to filter out less important background information e.g. find a specific item on a cluttered desk, pick out numbers in a word problem

Visual Memory: The ability to recall something seen, either immediately or when it is required later e.g. remember what was read, remember a particular symbol, remember how to use a calculator

Visual Sequencing: The ability to attend to and/ or recall correctly the order of letters/ numbers/ symbols/ words/ etc. seen or read (e.g. child might see 259 but might read or write 925)

Visual-Spatial Ability: The ability to perceive the location of objects, numbers and symbols and how they are placed in relation to each other (e.g. child needs to be able to align numbers vertically for addition or subtraction of multi-digit numbers; Trigonometry and Calculus require the ability to imagine an object rotating in space

Visual-Motor Processing: The ability to use feedback from the eyes to coordinate the movement of other parts of the body e.g. write within the lines or margins, copy from the board or a book

Visual Attention: The ability to stay focussed on visual tasks or stimuli. A student gets exhausted, restless or inattentive with the effort exerted in trying to process visual information

Teaching Tips	Visual Processing

- Minimise copying from textbook or board for students with visual processing challenges.
- Set the child up with a note-taking buddy so he/ she can concentrate on listening instead of struggling to record information.
- To help with spacing/ sizing, use thickly-lined, squared paper, dotted paper, graph paper or unlined paper depending on the student's preferences and needs.
- Place a number strip on the student's table so that he/ she can refer to it for correct numeral formation.
- Colour-code steps in maths problems.
- Use a multi-sensory approach when introducing and practising new concepts and skills (e.g. bendable pipe cleaners or writing in sand for forming numerals or shapes, using tangible cardboard clocks when learning to tell the time from an analogue clock).
- Repeat information in different modalities - say it aloud, demonstrate it, provide a handout, incorporate auditory information when possible.
- Encourage using a finger or ruler to guide the eyes during reading and to help the student keep his/ her place.
- Make use of tablets and other screens that can be enlarged. Zooming in on an image or piece of text can help reduce visual distractions and make it easier for a student to focus.
- Reduce/ eliminate clutter - clear the student's desk.
- Reduce visual distractions or position the student's desk away from them.
- Keep worksheets clear and simple - remove pretty borders.
- Incorporate specific activities to help build visual processing skills (e.g. hidden picture games such as 'Where's Wally?', odd-one-out, memory games, dot-to-dot activities.
- Consider using computer software for working on visual discrimination and visual memory.
- Encourage the student to take ownership, ask for assistance and self-advocate (e.g. remind self to pay attention to details, to use a highlighter or to check for errors).
- Give students a break. Include activities that don't require them to use their eyes. Plan lessons that require children to use other senses.
- Click **here** for NEPS Good Practice interventions for visual processing skills.

Kinaesthetic and Hands-On Learning

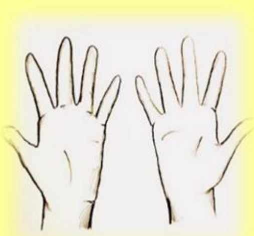


Fig.27 Hands-On Learning

Kinaesthetic learning takes place when students carry out physical activities rather than learning through listening or watching. This type of learning is a great way to learn maths concepts, especially for students who have learning challenges or different learning styles. The learning comes not just through the sense of touch, but through activities which allow students, for example, to make a model of a cylinder, to conduct an experiment with water using various containers to learn about capacity, to cut an apple into quarters or to draw a picture or diagram when solving a mathematical problem.

Executive Functioning Skills

Executive Functioning and Maths

Teaching Tips

Executive Functioning (EF) is an umbrella term for a range of interacting cognitive processes which enable us to perform or execute tasks - to plan, focus attention, remember instructions, control impulses, switch strategies and juggle multiple tasks successfully. In this section, we focus on those processes and associated skills which are important for the development of mathematical competence:

- Goal setting and planning - figuring out an end point and how to get there
- Flexibility of behaviour and thought - switching easily between approaches
- Organising and Prioritising - making decisions based on relative importance
- Accessing Working Memory - holding and manipulating verbal and non-verbal information in your head so you can make use of it
- Self-regulation, self-monitoring, managing your level of application.

Adapted from Meltzer, 2018

If you were a driver, EF skills would help you to turn on the ignition, know your destination, notice when you were running out of fuel, recognise when you were lost and identify a different route. When applied to mathematics, EF skills enable students to get started by trying a strategy, keep going, monitor their own progress and change strategies when things do not seem to be working out correctly.

The development of EF skills is dependent on many different factors and skills continue to develop through adolescence and early adulthood. For most students these skills develop naturally but some students need extra support for emerging skills. We can support and help strengthen their development through modelling, scaffolding and supportive relationships. We can adapt the environment (e.g. reduce time pressure, provide checklists) and teach students how to set goals, plan and prioritise, organise materials, shift approaches and monitor their engagement and performance.

Fig. 27 On track with Executive Functioning

It takes time to learn new strategies and practise them so that they become automatic and reliable. Time spent on teaching these skills in a maths class will:

- Ω Help students understand their strengths and challenges
- Ω Teach students how to learn
- Ω Promote motivation, focus effort and encourage independence
- Ω Empower students to take control of their learning
- Ω Increase confidence
- Ω Improve learning outcomes. (Meltzer, 2010)

At this time, the science is still emerging and the relationships between the various EF skills are not fully known. It is unclear, for example, if EF skills are entirely discrete. What we do know is that the best way to support the skills needed for effective maths progress is through really good teaching. One of the most important aspects of this effective teaching is ‘scaffolding’. Scaffolding is an approach promoted by Vygotsky (1978). Click **here** for more information. Scaffolding techniques such as breaking down skills into component parts, modelling and supported practice all have a significant impact on maths learning for those who struggle. Teachers, therefore, will want to create an environment where the required mathematical and EF skills can grow and develop in an evolving way. The following Teaching Tips may help to support such an endeavour.

	Teaching Tips EF Skills
Cognitive Flexibility	Ω Ensure that students have been taught many strategies for problem-solving. Ω Teach students to ask themselves: Is this problem similar to a problem I have seen before? Ω Teach thinking such as: *My first strategy did not work…maybe I should try a different one. Have I drawn a picture of the problem?* Ω Encourage students to talk to teachers & peers when stuck Ω Give opportunities to work independently, in pairs and in groups Ω Eliminate time pressure on students (at least initially).
Self-Monitoring and Self-Regulation	Ω Ensure that students know what they are required to do and the length of time it should take to do it. Ω Differentiate instruction (with individualised teaching strategies and classroom accommodations) for students with attention challenges. Ω Provide explicit checklists for assignments. Ω Help students to devise personal checklists so they recognise and monitor their most common errors/challenges. Ω Increase student awareness of EF strategies which work well for them. Ω Help students modulate emotional responses through strategies such as logical thinking, relaxation & positive self-talk. Ω Consider using incentives & rewards for starting on time, sustaining effort & completing tasks.
Goal Setting	Ω Set clear goals and objectives for your lessons and share with students. Ω Ensure students understand the purpose of each task. Ω Try to link maths activities to student goals and interests. Teach students to break down long-term goals into more easily achievable steps.

	Teaching Tips EF Skills
Working Memory	Ω Break instructions down into chunks Ω Use verbal & non-verbal reminders, prompts and cues Ω Use visuals Ω Use mnemonics/memory aids. Here are a few examples: o *5, 6, 7, 8 56 is 7 by 8* o < *and* > *The alligator has to open its mouth wider for the larger number* o *The value of pi (3.1415926): Count each word's letters in the question "May I have a large container of coffee?"*
Planning, Organising and Prioritising	Ω Use visual timetables. Ω Use *To-Do* notes written into a diary. Ω Display *How-to-do* lists with diagrams and instructions in classroom. Ω Encourage clutter-free workspaces. Ω Have checklists of equipment needed. Ω Use alarm on student's phone to act as a reminder (…may be more appropriate at home). Ω Support student organisation by providing work materials if necessary. Ω Encourage completion of subtasks when faced with a complex task. Ω Use different coloured highlighters for different types of information. Ω Teach strategic approaches to class work, homework and study.
	Ω Click **here** for more information on EF skills from Harvard University.

Please Note

At this stage, the evidence about the teaching of discrete executive functioning skills is inconclusive.

Good teaching involves bringing the various skills together and developing those skills in an interactive process. Teaching support is best deployed in teaching the maths skills rather than attempting to 'train' or 'teach' underlying discrete executive functioning skills.

Teachings from Neuroscience

Neuroscience and Education

Teaching Tips

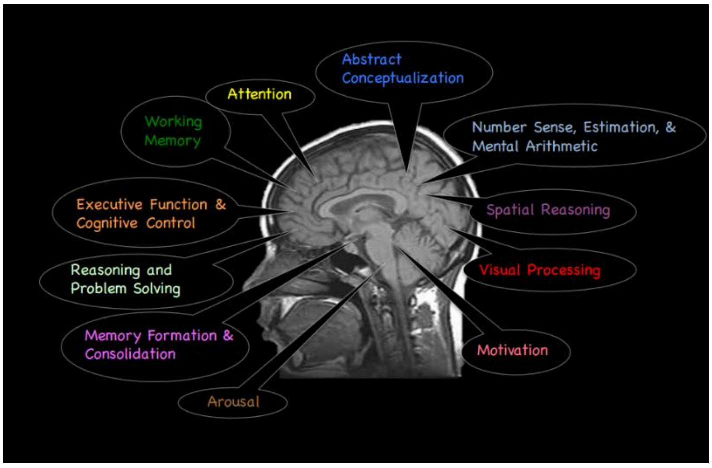


Fig. 28 The Maths Brain, Salimpoor (2016)

Neuroscience and Education

Our growing understanding of how the brain works has impacted on educational practice in recent decades. The demand for neuroscience-informed education comes from two directions, with neuroscientists emphasising the potential of their work to improve education and educators being keen to learn what neuroscience has to offer (Howard-Jones, 2014). One of the most useful functions of this emerging science is that it can reveal information that is *not* visible at the behavioural level. It has contributed to our understanding of the following:

- Ω Different brain areas involved in number fact retrieval, abstract thinking, imagery, spatial orientation, number sense and accurate counting
- Ω Differences in brain activity of people with dyscalculia
- Ω Connections between brain areas
- Ω How maturational and developmental changes impact on the brain's understanding of, and responses to, mathematical concepts and processes
- Ω How interventions and differing instructional approaches affect neural patterns
- Ω Impact of maths anxiety and of beliefs and mindsets on neural activity patterns.

Teaching Tips *from* Neuroscience

- Help students to understand that the brain can change through effort and practice and that intelligence, like a muscle, grows stronger with exercise.
- Read a short article about Growth Mindset by Carol Dweck (2015) **here**. A fixed mindset is a belief that your intelligence, abilities and talents are fixed traits. A growth mindset is a belief that your abilities can be developed through dedication and hard work. Neuroscience confirms that a growth mindset leads to more activity in the brain's thinking parts, to forming more neural pathways and to quicker learning of new information.
- Click **here** for Jo Boaler's infographics about maths mindsets.
- Use hands-on materials for building mathematical understanding of concepts. This facilitates multisensory learning, aiding both understanding and memory.
- Encourage finger representation for number concept development and arithmetic (Bafalluy and Noel, 2008). There is evidence that fingers have a special place over and above concrete materials.
- Separate learning sessions in time (spacing), as opposed to massing them together, as this has been shown to improve learning performance (Rohrer and Taylor, 2006).
- Neuroscientific research confirms the value of supporting students to construct their own knowledge and solutions through game playing and hands-on learning. A constructivist approach works better than direct instruction through lecture, practice work, homework and exams (Burnett, 2010.)
- Physical exercise has been shown to increase the efficiency of neural networks in learning (Diamond, 2012).
- Consider using games created using learning from psychology and neuroscience. Read more **here** in an article called "From Brain to Education" (Butterworth et al, 2011).
- Maths learning needs an environment that is flexible in content and pace for students' current needs and zone of proximal development (definition of the latter on page 15).

Section D

Page

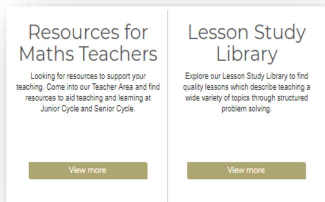
Current Teacher Supports

Resources and Professional Development Opportunities

The Professional Development Service for Teachers is a support service for **primary and post-primary** staff, offering professional learning opportunities to teachers and school leaders in a range of curricular areas, including mathematics.

The PDST website contains resources, publications, school self-evaluation tools, lists of useful websites and apps, and video footage of good practice. The PDST offer *Maths Recovery* and *Ready, Set, Go - Maths* training for teachers of young children. They offer one-day seminars and workshops on mathematics topics (e.g. Problem-Solving, Mental Maths) in Education Centres throughout the country. The majority of these events are booked through the **central applications system.** PDST also offer bespoke in-school support sessions to teachers across all areas of teaching and learning in maths, from early years through to Leaving Cert. They support teachers' inclusive practice in relation to team teaching for maths. Apply at **pdst.ie/schoolsupport**. If you have any difficulties registering, then email schoolsupport@pdst.ie.

The PDST Post Primary Mathematics Team (previously known as *The Project Maths Development Team*, then *The Maths Development Team*) supports all **post-primary** Mathematics teachers. The team of experienced teachers provides professional development support to post-primary teachers through workshops, *Lesson Study* materials, school visits and online resources.



Visit **www.projectmaths.ie** for more information or apply for in-school support at **pdst.ie/schoolsupport.** Click **here** to see an expert teaching maths.

The **Special Education Support Service,** part of the National Council for Special Education, develops and delivers professional development supports for school personnel working with students with special educational needs in primary, post-primary, special schools and special classes. Access SESS support through its **online application** system. The SESS offers telephone advice, school visits, and staff in-service courses. Check out the SESS **Curriculum Access Tool** (*CAT*) for primary school learners with mild, moderate and severe general learning difficulties, or the SESS **Tips for Teaching Learners with Dyscalculia.**

The **Educational Research Centre** website **www.erc.ie** includes research reports on Ireland's national and international mathematics performance. These reports contain ideas and recommendations on teaching mathematics e.g. **PISA Maths: A Teacher's Guide.** The ERC provides Drumcondra maths tests for primary and post-primary schools in paper and digital formats.

The National Council for Curriculum and Assessment **NCCA website** contains curricula, research reports, assessment toolkits and videos of good practice in maths. The NCCA and its educational partners are developing a new primary mathematics curriculum. Read a background paper **here.** **Research Reports 17** and **18** provide valuable up-to-date information about mathematics in early childhood and in primary education.

The PDST **Junior Certificate School Programme** (**JCSP)** is targeted at Junior Cycle students at risk of early school leaving. JCSP at **www.jcsp.ie** offers **mathematics resources** and professional development support to post-primary schools in the Delivering Equality of Opportunity in Schools (DEIS) initiative. The resources are available for purchase to schools outside this initiative.

The US National Council of Teachers of Mathematics provides guidance and resources to implement research-informed and high-quality teaching supporting every student's learning in equitable environments. Visit their **website** for information.

EPI-STEM, the national centre for STEM Education is based at the University of Limerick. EPI-STEM aims to strengthen STEM education research, inform STEM education policy and promote STEM in **primary** and **post-primary schools,** and in the community. Its website contains research reports and links to maths resources.

The **What Works Clearinghouse** (WWC) reviews research on different US programs, products, practices and policies. It helps teachers and school leaders in making evidence-based decisions.

Interventions and Initiatives

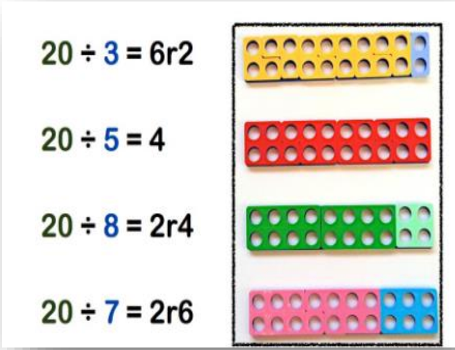
Primary School

Maths Recovery (Wright, 2003) is a widely-used intervention based on very detailed pupil assessment. Its framework for individual, group or class-based instruction is suitable for pupils with or without maths challenges. The Maths Recovery Programme is one of the Department of Education and Skills DEIS (Delivering Equality of Opportunity in Schools) initiatives to improve numeracy outcomes. Maths Recovery develops children's knowledge of number words and numerals, conceptual place value, addition and subtraction to 100, multiplication and division and written computation. It emphasises mental calculation and relational thinking, encouraging students to see relationships between numbers rather than to follow rules. In Ireland, it is usually used with students in First Class and it involves 1-1 daily sessions of 25 minutes for teaching cycles of 12 to 15 weeks' duration.

Mata sa Rang and **Maths Blast** are in-class numeracy teaching approaches using Maths Recovery strategies. The assessment tasks identify pupils' strengths and needs (Cull, 2018). Local, experienced and practising Maths Recovery teachers provide training in Education Centres nationwide. Click **here** to see the Maths Recovery approach in action.

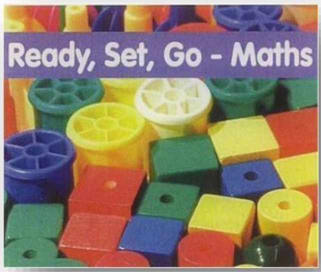
Number Worlds (Griffin, 2005) is an early intervention programme used to close the number knowledge gap between children in schools in low-income, high-risk communities and their more affluent peers in Massachusetts (Griffin and Case, 1997) and in Dublin (Mullan and Travers, 2007). The intervention involves whole class teaching and scaffolded small group work. It emphasises counting and language skills to help children gain a number representation similar to a *mental counting line*. Number Knowledge and Level Placement Tests measure students' conceptual knowledge and pinpoint instructional start points. There are ten levels in the programme (Levels A to J) which can be used from pre-school right through primary school (and beyond) - usually one level per school year.

Numicon is a multi-sensory maths teaching programme for children aged 4-7 years (and older students with special educational needs) which uses Numicon Maths Shapes in practical teaching activities. The Maths Shapes give learners insight into number values and relationships, differently to that given by written numerals. The programme teaches number concepts and more complex concepts such as multiplication, division and fractions. Learners develop their own mental imagery as they combine and compare shapes and use arithmetic in practical activities. Nye, Buckley and Bird (2005) found Numicon to be effective in teaching children with Down Syndrome. Skevinton (2016) also found that Numicon was useful with older primary school children with number concept challenges.



Catch-Up Numeracy is based on the Numeracy Recovery intervention programme developed by Dr. Anne Dowker, University of Oxford, in 2001. It is a Teaching Assistant - led programme involving two 15-minute, one-to-one teaching sessions weekly. This intervention breaks numeracy down into ten components. It is funded by the **Education Endowment Foundation** and is targeted at **6 to 14-year olds**. Training comprises three half-day training sessions and is offered in the UK and Northern Ireland.

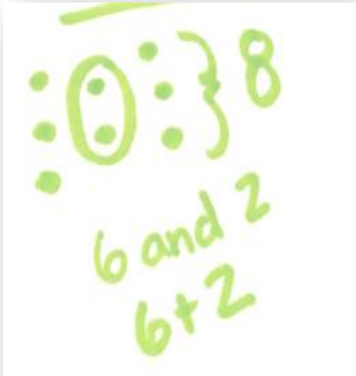
Ready, Set, Go - Maths



Ready, Set, Go - Maths was developed in Northern Ireland between 1999 and 2001 by Eunice Pitt. This is a programme for teachers of infant classes which focuses specifically on the development of early number skills and concepts. Research (unpublished at time of printing) showed Ready, Set, Go - Maths to be an effective means of including children with SEN in a mainstream junior infant classroom (in Dublin) over an eight-week intervention period. Training is available from PDST.

JUMP (Junior Undiscovered Math Prodigies) is a Canadian class-based programme of confidence-building, guided practice, guided discovery, continuous assessment, scaffolded instruction, mental maths and conceptual understanding. Read more **here** about an evaluation of JUMP in two Education Centre catchment areas during 2013/14. The JUMP meaning of guided discovery is more didactic than that recommended in the Primary School Mathematics Curriculum.

Number Talks are 5-15-minute conversations around purposefully-crafted computation problems. The talks get children *thinking* and *talking about their thoughts* when presenting and justifying solutions to computation problems. This programme requires a safe and risk-free environment, with a culture of acceptance of all ideas and answers, regardless of errors. You can see Number Talks in action in Dublin **here.** Training is available from **PDST.**



Paired Mathematics and Mathematics for Fun are initiatives from the Home School Community Liaison Scheme. They involve parents engaging in mathematics games with children in the classroom.

Read more about interventions in Dowker (2009) **What Works for Children with Mathematical Difficulties.**

Be Careful

Ensure that an intervention programme is not just an “add-on”.

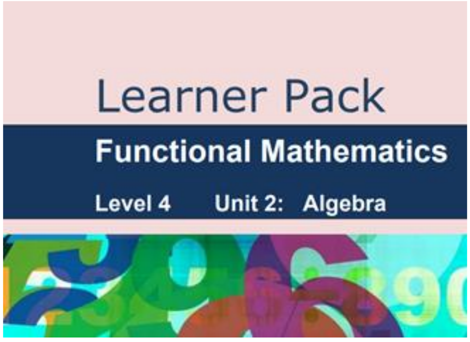
Readymade programmes tend to dictate how each topic is taught regardless of the student’s particular challenges or learning style.

Intervention programmes should identify mathematical difficulties through detailed initial assessment and subsequent ongoing diagnostic observations.

(Haseler, 2008)

Secondary School

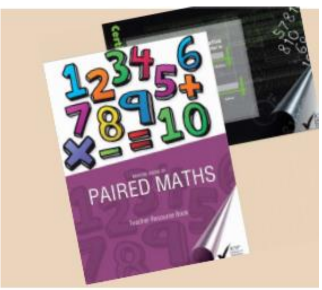
Project Maths: This professional development support to post-primary maths teachers is available from the PDST Post Primary Maths *Team*. You will find links and information about Project Maths in the *Current Teacher Supports* section.



Functional Mathematics, Application of Number and **Functional Mathematics Learner Packs** are resources to support students in developing knowledge, skills and competence in maths and in working towards FETAC/QQI Qualifications at Levels 3 and 4. These resources include Tutor Guides and Practice Sheets and they were developed by a team from the National Adult Literacy Agency (NALA) and the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL).

Junior Certificate School Programme (JCSP) Mathematics Initiatives

The JCSP Mathematics Initiatives enable schools to acquire age-appropriate experiential resources and games for mathematics and numeracy. The materials help to develop creative approaches to teaching maths to JCSP students.



Cross-Age Paired Maths: This strategy involves JCSP students training as tutors and working with 3rd class primary school students over a six-week programme. The strategy aims to enhance both groups' maths skills, competencies and confidence. Support materials and implementation guidelines are provided.

Number Millionaire is a numeracy quiz where individual students are challenged to identify the correct answer to twelve arithmetical questions. The quiz follows the "Who Wants to be a Millionaire?" format. Questions and answers are provided to participating schools.

Maths Laboratory consists of standardised, graded, colour-coded and differentiated work cards. Individual students' needs, prior knowledge and competency levels determine the appropriate learning route through the programme.

Hand–held Gaming Devices: Teachers are asked to explore and choose the most appropriate handheld device(s) currently available and then choose the most suitable software e.g. **Challenge Me Maths Workout, Personal Trainer Maths, Brain Age Express Maths, Maths Play, Maths Blaster, etc.**

Maths Games Initiative: This initiative provides opportunities for teachers to source and acquire age-appropriate maths games and/or maths activity packs. It encourages use of these resources to develop mathematical and numerical understanding among JCSP students. Teachers may also opt to plan, construct and develop their own maths games and activity packs.

An Invitation

We are aware of other commercially-available interventions used in Ireland e.g. *Mathletics* and *The Power of 2*. We would like to hear from you if you have evaluated these or any other interventions not included in this section, so that we can share their effectiveness in revisions of this document.

Please email neps@education.gov.ie with such information with the word **maths** in the subject line. Thank you.

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Appendix 1 List of Maths Tests

Before you purchase a test, think about:

- Ω The purpose of the assessment
- Ω What aspects/ areas of maths do you want to measure?
- Ω What tests are readily available in the school?
- Ω Would other methods of assessment be sufficient or superior?

When you have decided to purchase a test, think about:

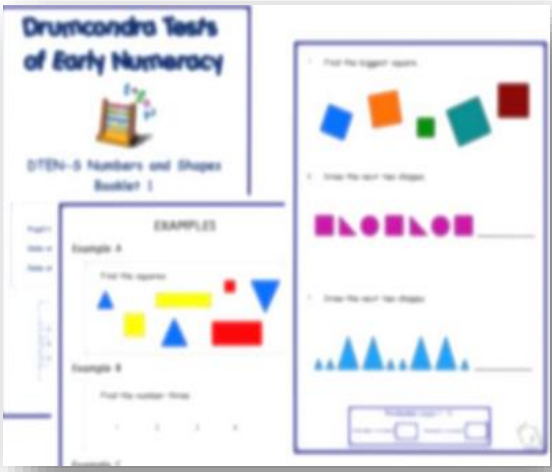
Ω Age-group Ω Individual or group Ω Cost Ω Guidance about follow up

Ω Length of time needed to give test Ω Standardisation Country Ω Reliability

Ω Scoring - manual or online Ω Qualifications needed for administration

Ω Alternative versions for re-testing Ω Validity Ω Research Evidence

The Drumcondra Tests of Early Numeracy (DTEN), 2008



End of Senior Infants/ Beginning of First Class

The Drumcondra Tests of Early Numeracy consist of two tests - the DTEN-Screening (DTEN-S) and the DTEN-Diagnostic (DTEN-D). They are based on the Irish Primary Mathematics Curriculum. The tests complement a teacher's observations and assessments of students' performance and highlight particular difficulties for focussed instruction. The DTEN-S identifies students at risk of numeracy difficulties. If needed, further diagnostic assessment and targeted instruction can be implemented. DTEN-S is divided into three parts: 1. pre-number, 2. numeration, 3. addition and subtraction. Teachers can administer this 40-minute test to **small pupil groups** (10 or fewer). The DTEN-D is a criterion-referenced diagnostic test. It is used with children who have low scores on DTEN-S. DTEN-D takes approximately 45 minutes to administer individually. This test has 15 tasks covering the three areas of pre-number, numeration, addition and subtraction. The DTEN manual summarises recent research on early number development and advises on early numeracy interventions.

The Drumcondra Primary Mathematics Test - Revised (DPMT- R), 2005

End of First Class - End of Sixth Class (Six Levels)

The DPMT-R comprises norm-referenced mathematics achievement tests developed for Irish primary school **group** administration. English and Irish language versions are available. The tests contain six levels, most with parallel forms. These revised tests reflect the 1999 Primary School Mathematics Curriculum and cover Number/ Algebra, Shape and Space, Measures and Data. Administration time is between 90 and 120 minutes and calculators are permitted. The administrator reads Levels 1 and 2 aloud.

The Drumcondra Post-Primary Test -Mathematics (DPPT- Maths), 2016

Final Term of Second Year

The Drumcondra Post-Primary Mathematics Test (DPPT-Maths) is a norm-referenced curriculum-based assessment for students finishing Second Year. It complements the

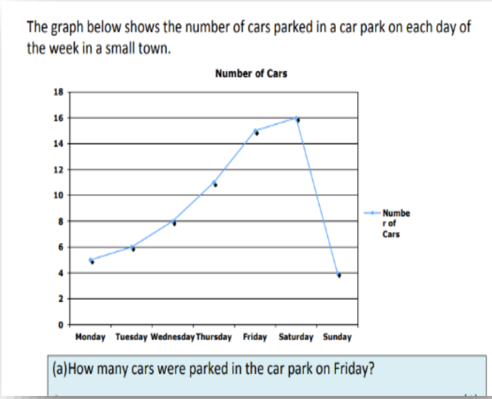
Junior Certificate Mathematics Syllabus and has four content areas: Statistics and Probability, Geometry and Trigonometry, Number and Measurement, Algebra and Functions. Administration time is 70 minutes for **individuals** and **groups.** The DPPT-Maths *paper* version has 2 parallel forms while the *digital* version has four.

Standardised Irish Graded Mathematics Attainment Tests (SIGMA T), 2007

Beginning of First Class - End of Sixth Class (Five Levels)

The SIGMA-T series of mathematics attainment tests is standardised for Irish primary schools. The series' five levels are based on the Mathematics Curriculum. They cover Number, Measurement, Geometry, Algebra, Data and Statistics. Teachers can deliver the SIGMA-T Tests **individually** or in **groups.** Parallel forms are available. Administration time is between 70 and 120 minutes. The SIGMA-T is available in Irish.

PDST First Year Maths Competency Tests



The First Year Maths Competency Tests available here follow the 6th Class Mathematics curriculum. Schools can choose from two versions. Accompanying spreadsheets assist in analysing students' performance. First Year Maths Competency Tests are not standardised assessments. Teachers can administer these formative assessment tools early in First Year to **individuals** or **groups**. They can then identify individual or collective weaknesses and put appropriate interventions in place. Administration time is 40 to 45 minutes.

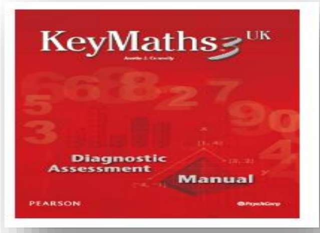
PDST Senior Cycle Maths Competency Tests

Transition Year or Fifth Year

Based on the Junior Certificate Syllabus, these Ordinary- and Higher- Level tests help identify weaknesses and relevant interventions.

Key Maths 3 U.K, 2014

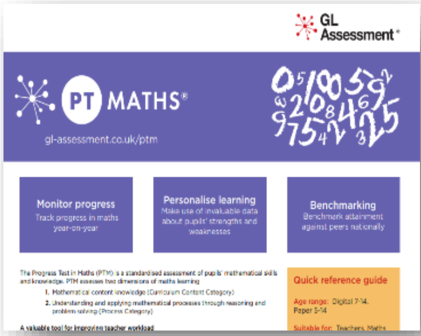
Age Range: 6-16 years



This is a norm-referenced measure of key mathematical concepts and skills: basic concepts, operations and applications. **Individual** administration time is between 30 and 90 minutes.

Progress Test in Maths (PTM), 2015

Age Range: 5-14 years



The PTM, 2015, is a standardised assessment of mathematical skills and knowledge, based on the UK 2014/ 2015 Maths Programmes of Study. Available in paper and digital format, group administration time is 60 to 75 minutes. A formative and summative assessment, the PTM is useful when starting and finishing an academic year.

The PTM assesses two dimensions of mathematics:

1) Mathematical content knowledge – curricular areas Number, Shape, Data, Algebra

2) Understanding and application of mathematical processes through reasoning and problem-solving.

Access Mathematics Tests (2008)

Test 1 Age Range: 7-12 years

Test 2 Age Range: *11-16+ years*

Teachers can deliver these UK-normed standardised tests individually or in groups. Both manual and digital versions have parallel forms.

Wide Range Achievement Test, Fifth Edition (WRAT 5), 2017

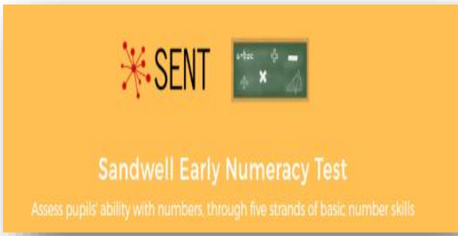
Age Range: 5-85 years

The WRAT 5 maths assessment is a computation assessment measuring counting ability, number identification and oral and written maths computations. Written calculations cover arithmetic, algebra, geometry, and advanced operations. It is a US norm-referenced test. WRAT 5 has two alternative forms, allowing pre- and post-testing of an intervention. **Individual or group** administration time takes between 15 and 45 minutes. Paper and digital versions are available.

Sandwell Early Numeracy Test-R (SENT-R), 2011;

Age Range: 4-8 years

Sandwell Early Numeracy Test- Key Stages 2 and 3 (SENT-KS 2,3), 2012; Age Range: 8-14 years



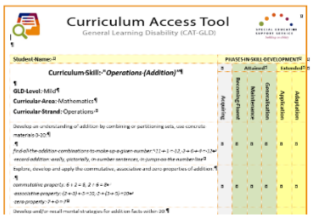
SENT-R and SENT-KS 2, 3 are UK norm-referenced tests, with parallel forms, assessing five basic number skills: number identification, oral counting, object counting, value and computation, and language. Administration is **individual** and takes 10 to 30 minutes. An online marking tool analyses data and generates diagnostic reports. Sandwell tests are used extensively in UK schools.

Mathematics Assessment for Learning and Teaching (MaLT, 2009)

Age Range: 4-15 years

MaLT, 2009, is a series of UK standardised maths tests. Three Teacher Manuals match the 3 Key Stages. Class test administration takes 45 minutes and the tests can be administered at any time in the school year. It is useful for screening, monitoring, tracking progress and individual diagnostic profiling. Computer-based scoring and error analysis yield diagnostic and summative reports. Age 4 to 7 tests are administered orally.

The Curriculum Access Tool for Students with General Learning Disability (CAT-GLD) Click here for the **SESS CAT Maths Tool**, useful for students with learning difficulties



Basic Number Screening Test, 4th Edition, Bill Gillham et al, 2012

***Age Range*: 6-12 years**

This quick standardised UK assessment focuses on children's understanding of number and number operations. Its oral administration takes 30 minutes to deliver to **class groups** or **individuals**. Parallel forms enable re-testing.

Basic Number Diagnostic Test, 3rd Edition, Bill Gillham, 2001

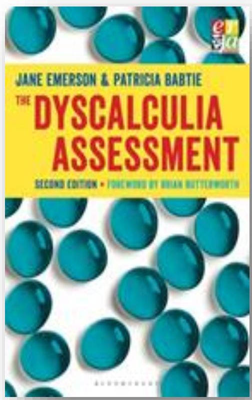
Age Range: 5-7 years

This test, allied to the UK curriculum, is criterion-referenced (but with approximate norms). Administration time for individuals is 15-25 minutes. It is used for early **individual screening** of all children during the first two years of primary school or alternatively with those children who appear to be experiencing number difficulties. It yields detailed information on a pupil's basic understanding and skill development. The accompanying manual provides intervention activities. The test can be used two-to-three times a year to chart progress and revise teaching objectives.

The Dyscalculia Assessment, Second Edition, **2013**

Age Range: 5-12 years

The Dyscalculia Assessment investigates a student's numeracy abilities. It informs personalised teaching programmes for individuals or small groups with number difficulties. Individual administration time is 60 minutes. Alternatively, the Dyscalculia Assessment can be conducted over several sessions.



More Trouble with Maths, Steve Chinn, 2016

Age Range: 5-59 years

This book contains norm-referenced achievement tests that can be photocopied. It also includes tests of basic number facts, thinking style and maths anxiety as well as a dyscalculia checklist. '***More Trouble with Maths'*** and '***The Trouble with Maths***', both by Steve Chinn, offer practical ideas and strategies.

Appendix 2 Checklists

Basic Needs Checklist

PUPIL'S NAME	DOB	CLASS	DATE

Headings under which to consider a pupil's basic needs:

- [ ] Physiological needs e.g. does the pupil have adequate food, warmth, housing, etc?
- [ ] Safety needs e.g. does the pupil need physical or psychological protection?
- [ ] Belonging needs e.g. does the pupil have close family and friends, feel part of his/her class?
- [ ] Esteem needs e.g. does the pupil receive respect and positive feedback from others and does he/ she respect others and self?

Possible Actions

Classroom Support Checklist

PUPIL'S NAME	DOB	CLASS DATE
Parents Consulted		
Information from previous school/preschool, or previous class teacher		
Hearing		
Vision		
Motor Skills		
Medical Needs		
Basic Needs Checklist completed		
Assessment of learning – screening, attainments tested, if appropriate		

School Support Checklist

PUPIL'S NAME	DOB	CLASS	DATE

GENERAL INFORMATION	CHECKED (YES/NO)	COMMENTS
Parents Consulted		
Information from previous school/preschool gathered		
Hearing		
Vision		
Motor Skills		
Medical Needs		
'Basic Needs' Checklist completed		
Assessment of learning – screening, attainments tested, if appropriate		
Observation of learning style/ approach to learning		
Observation of behaviour		
Interview with pupil		
Classroom work differentiated		
Learning environment adapted		
Playground/school environment adapted		
Informal consultation with outside professionals		
Direct input from supporting teacher/s		
Other interventions in place in school		
ACTION NEEDED		