THE INSTITUTION OF ENGINEERS OF IRELAND

Flood Estimation following the Flood Studies Report

C. Cunnane B.E., Ph.D., M.I El.
M.A. Lynn M.E., B. Sc., C. Eng., F.IEI.

Presented at a meeting of the Civil Division on 9th February 1975 at 5.30pm in the Engineering School U. C.D.

Synopsis

The Flood Studies Report was published in April 1975, and dealt with an investigation carried out since 1970 at the Institute of Hydrology, Wallingford and the Meteorological Office, Bracknell. The Irish Office of Public Works and the Meteorological Service participated in the studies and supplied data on floods and rainfall in Ireland for inclusion in the analyses. The results are accordingly applicable in this country and provide much improved bases for flood estimation. They can in fact be said to supersede methods in use heretofore. In the paper the results are discussed, the principal methods are introduced and the application of some is illustrated by practical examples. In particular, methods are described for evaluating rainfall input in flood estimation, predicting flood peaks, synthesising flood hydrographs and estimating the maximum flood at a site. The methods deal with two situations (a) where records are available and (b) where none exist.

*References to specific parts of the Flood Studies Report are given as footnotes throughout the paper.

The Irish analyses have also been published separately as Extreme Rainfalls in Ireland by J. Logue (1975), Technical Note No. 40, Meteorological Service, Dublin.

Introduction

The Flood Studies Report [(1)] published in March 1975, is based on extensive research and analysis of available hydrometric and rainfall records in Britain and Ireland. It includes a volume each on hydrological, meteorological and flood routing studies as well as a volume containing the data used and a volume of maps for applying the results derived in the analyses.
The investigation that led to the Report was an event of historical importance and involved the employment over a number of years of considerable resources and expertise. The amount of data used also shows the thoroughness of the studies undertaken. The flood frequency studies used some 5500 record years from 430 British gauging stations and 1700 record years from 112 Irish sites (Map 1); unit hydrograph studies used 1500 rainfall-runoff events on 140 catchments. Before the selection of gauging stations, sites were inspected and their records and rating curves assessed. For Britain 600 daily rainfall records of average length 60 years, 6000 daily records for the decade 1961 -70 and 200 autographic rain gauge records were subjected to depth-duration-frequency analysis; for Ireland the records of 330 daily and 12 well spaced autographic gauges were analysed* (Map 2).

Availability of the report should make for significant changes and improvements in flood and rainfall estimation practice. For natural catchments, flood estimation should be based on the methods recommended in Volume I of the Report, which means the rejection of traditional ones such as the Rational Method and its variations. Rainfall estimates should be based on the methods of Volume II for both natural and urban catchments and formulae such as those of Bilham and the Ministry of Health are superseded. In urban drainage problems, the Road Research Laboratory [(2)] and the Rational Methods [(3)] may continue in use with the rainfall methods of Volume II. The methods recommended are not always simple or easy to apply but it is implicit in the Report that simple flood estimation methods, which are also reliable, are not attainable. The average interval between exceedances of a specified flood magnitude, Q, is called its return period, T, and has a reciprocal relation with probability of exceedance. When a design flood value is required, the designer must establish its required return period in consultation with his client. This depends on the risk that can be tolerated and on the relationship between costs and benefits for different standards of flood immunity. A risk-free design is provided only when the estimated maximum flood is specified.

When the design return period is specified the corresponding discharge value is obtained from the Q-T relation, the method of estimation depending on the amount of data available at and near the site. The best estimates are obtained when the parameters of the Q-T relation are calculated from recorded data. If records are not available the parameters have to be obtained from catchment characteristics (Section3.1.1 and 3.1.2), but consideration of the standard errors shows that such estimates have the worth of about one year's data. Consequently, the extra work and inconvenience of providing a record and analysing the data is absolutely necessary in calculating a design flood for works of any importance.

Estimating the Q-T relation may be approached in two ways (i) by statistical analysis of flood flows and (ii) using a unit hydrograph to transform rainfall of specified frequency into runoff. In either case the procedure depends on the amount of data, if any, available at or near the site. Local knowledge should be taken into account in particular applications.

Scope of Paper

The aim of this paper is to bring to the attention of Irish engineers the results in the Report of most immediate concern. It describes the background and events leading to the Flood Studies and summaries the findings of the meteorological studies. There is a discussion of climatic indices and catchment characteristics leading to the development of empirical flood formulae. Later sections deal with the recommended flood estimation methods viz. statistical estimation of flood peaks (Section 4), flood estimation by unit hydrograph (Section 5) and the extension of this to estimating the maximum flood (Sect.6)

(Some important material in the Report dealing with matters such as flood routing* and extension of short records† could not be covered.)

1. Background to Flood Studies

The Flood Studies can be said to have originated from the 1933 Interim Report of the Institution of Civil Engineers (I.C.E.) Committee [(4)] and its extended reissue of 1960[(5)]. These standard references however, specified only large floods for spillway design purposes in upland areas and did not cater for estimation of a flood of given return period which is required when the relationship between costs and benefits for different standards of flood immunity is being studied.

A paper to this Institution [(6)] in 1971 described the unsatisfactory methods of flood estimation that were in use in this country. These methods made little use of the large amount of available data - hydrometric and meteorological - and there were obvious needs for their research and analyses.

Following an I.C.E. symposium [(7)] in 1955, a further report [(8)] was issued in 1967 setting down terms of reference for a comprehensive investigation of floods and extreme rainfalls in Britain. The Natural Environment Research Council agreed to finance the project and in 1970 the Flood Studies Team, under Dr. J.V. Sutcliffe, was assembled at the Institute of Hydrology, Wallingford and a meteorological group, under Mr. A.F. Jenkinson, was formed at the Meteorological Office, Bracknell.

Irish involvement in the Flood Studies was initiated by an approach, through Prof. J.E. Nash of U.C.G., to the Irish Committee of the International Hydrological Decade to have included in the study the considerable number of long records available in Ireland. The Office of Public Works agreed to cooperate and set up a hydrological unit for this purpose and the Irish Meteorological Service also agreed to carry out analyses of rainfall data similar to those of the British Meteorological Office. It proved a very satisfactory exercise in international cooperation and has had the effect that the results of the studies are fully applicable in this country. The work on the Flood Studies was guided by an interdepartmental Steering Committee and lasted almost five years. Its objectives were:

(i)To provide a means of estimating the Q-T relation for flood peaks at sites with or without hydrometric data. This relation is necessary where economic considerations require that some periodic flooding be permitted. The peak alone is sufficient for some design problems, as in determining the waterway required under a bridge.

(ii)To provide a means of estimating the flood hydro-graph, of given peak return period, for use in circumstances where flood volumes and storage attenuation are involved e.g. for routing through reservoir or channel storage.

(iii)To provide an estimate of the maximum possible flood. This is a frequent requirement in reservoir and spillway design and involves the volume and shape of the flood.

(iv)To provide rainfall depth-duration-return period relationships over any area, to enable estimates of storm depths and profiles for all return periods within the range of all possible rainfalls to be made.

2. Results of Rainfall Studies[††]

Rainfall estimates, derived from analyses of the extensive rainfall records already described, are now possible that are much more reliable than heretofore and that meet all the normal needs in flood estimation regardless of the method used. They enable the rainfall depth (DR_T) of any duration (D) and return period (T) to be determined for any point or over any area in Britain and Ireland.

2.1 Point rainfall estimation (DR_T)

The analyses were based on rainfalls of 5-year return period (R5) with durations 2-day and 60 minutes and the results were then related to rainfall depths for other return periods and durations. The 5-year quantity, the basic building brick in the scheme, was used because of observed discontinuities at lesser return periods in probability plots of rainfalls. As a result, estimation of long-term rainfall R_T, is more reliably based on R_T/R5 than say R_T/R2. From this viewpoint, R10 would be preferable to R5 but the latter could be estimated more accurately at the very large number of short record stations. The 2-day duration was used because many important rainfall events are cut in two by the 9 a.m. observation of daily read gauges that were widely used in the analyses. The Report provides detailed maps of 2-day R5 and r = 60-min. R5/2-day R5. (For Ireland, see maps 2 and 3). From these, DR_T is estimated in two steps as follows:

First the 5-year return period rainfall for the required duration (DR5) is computed from 2-day R5 values, using the ratio r_D = DR5/2-day R5 which is related to r for durations 1 min. to 48 hours, as shown in Fig. 1, and to average annual rainfall, R, as in Table 1, for durations 48 hours to 25 days.

Then the T-year rainfall is obtained by multiplying DR5 by the growth factor R_T/R5, given in Fig. 2. These factors were derived from growth curves** given by plotting rainfall against return period for stations with R5 within a specified range (say 40 to 50 mm). It was found that for each R5 range a single curve could represent these plots, even when they included rainfall of various durations, provided their R5 values were within the specified range. The result was a family of growth curves, independent of duration, from which the R_T/R5 relationships were obtained. A high level of reliability is claimed for the resulting R_T estimates, up to return periods several times the average length of the records analysed.

2.2 Areal Estimate of DR_T.

The areal reduction factor (ARF), by which point rainfall of given frequency is multiplied to give corresponding areal rainfall, is shown in Table 2. This factor was found not to vary significantly with location or return period but to increase for a specified area (A) with increasing duration (D), and to diminish with increasing area when duration is specified. These results are, of course, in accordance with experience from which it would be expected that persistent rain (large D) would have greater areal uniformity and that full intensity might not be widespread for rain of short duration.

2.3 Storm profiles

These were derived to enable rainfall, for a given return period and area, to be suitably distributed over its duration (D). The need for an accurate distribution is particularly obvious when D is large (say several hours) and much past attention has been devoted to finding methods which would give design rainstorms that properly reflected the flood producing capacity of rainfall of given duration and frequency.

	Average Annual Rainfall, mm/100			
Duration (D)	**8 - 10**	**10 - 14**	**14 - 20**	**20 - 28**
48 hours	1.06	1.06	1.06	1.06
72 hours	1.19	1.2	1.21	1.22

96 hours	1.33	1.36	1.38	1.4
192 hours	1.79	1.89	1.95	2
25 days	3.16	3.56	3.84	3.91

Table 1: Estimated values of r_D (the ratio DR5/2 day R5) for durations between 48 hours and 25 days.

* Volume III, † Volume I, chapter 3, †† Volume II, ** Volume II, chapter 2.3

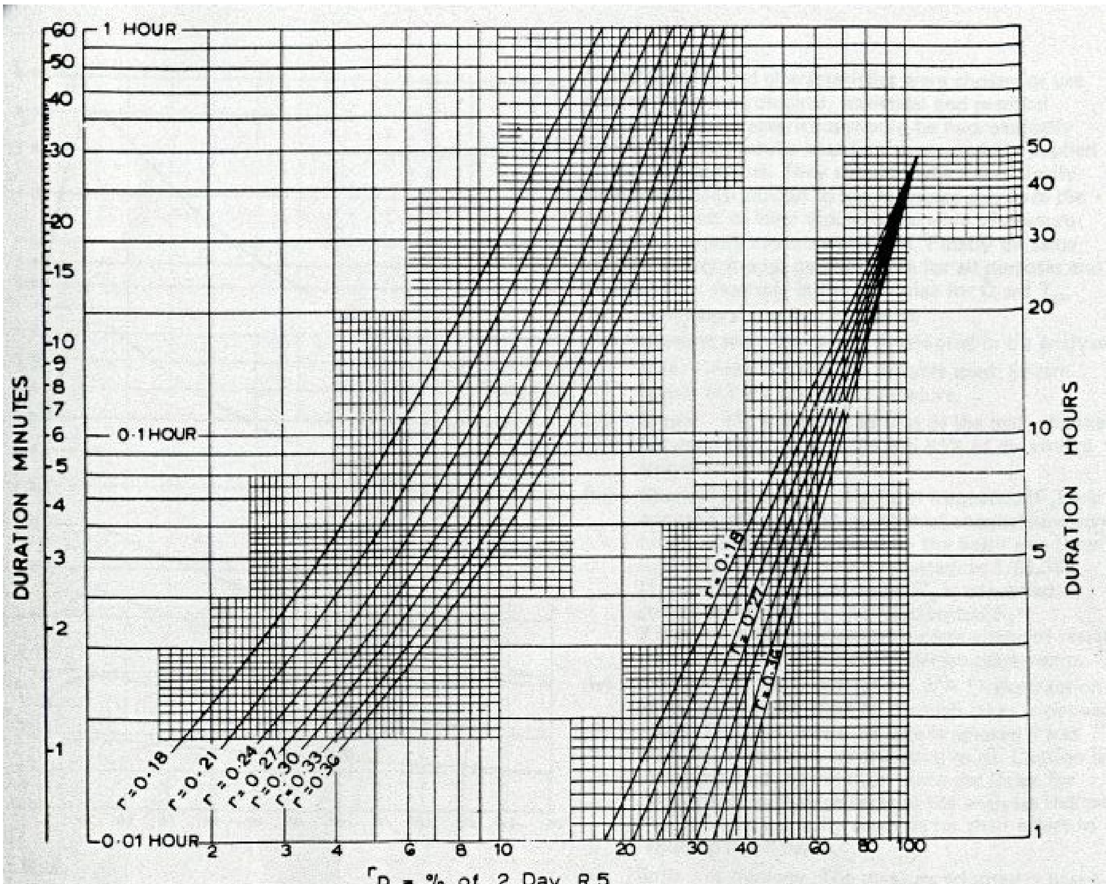


Fig. 1

The profiles recommended were derived from a large number of major storms, with winter and summer events examined separately; the primary analysis was based on 24-hour duration. To compare profiles the centre of each storm was defined to be the midpoint of the shortest duration giving 50% of its rainfall and individual storms were then grouped into quartiles, based on their peak-ness, i.e. on the ratio of their central 5-hour rainfall to the 24-hour rainfall. Mean profiles for each quartile then gave the % of total 24-hour rainfall in each hour and hence cumulative % of rainfall was related to cumulative % of duration (D); by interpolation, similar relationships were derived for other percentiles of storm peak ness. The result is a set of profiles, symmetrical in shape about the storm centre, from which a design profile can be selected, whose peak ness is exceeded in any desired % of storms. For flood estimation in rural conditions the Report recommends the use of the 75% winter storm profile shown in Fig. 3. Results for summer storms, due to thunderstorms rather than continuous rain, show sharper peaks. Seasonal effects also masked the geographical variation in storm profiles, which could not be satisfactorily quantified; the recommended profiles are therefore applicable to all areas (but caution is recommended in using summer profiles for mountainous areas).

Further analyses evaluated the effects on storm profile of variations in storm duration (60 min. to 4 days), return - period (up to 30 years) and of areal distribution. The results showed no significant changes, so the recommended profiles are applicable for design storms regardless of duration, return period or area involved.

2.4 Estimated maximum rainfall

In certain design situations, for example in reservoir and spillway design, estimates of the maximum flood are required and for this the Flood Studies Report provides estimates of maximum rainfall. The primary research was based on major storms of 2 and 24-hour durations, using the concept of storm efficiency. This is the ratio of actual rainfall to the amount of precipitable water in the representative air column at any instant during the storm and its maximum value in major storms at 60 stations was derived from recorded maximum dewpoint values. The amount of rainfall that would have fallen, if the maximum storm efficiency observed among all storms had been reached, was computed for each storm examined. These estimates of R_{max} for 2 and 24-hour durations were mapped and are shown for Ireland in Map 4. (It will be noted that the 24-hour values for all parts of Ireland are close to 300 mm (12") and 2-hour values are roughly half of that). These rainfall maxima were also compared with estimates given by an envelope of all known data of extremes used in the derivation of growth curves (Section 2.1); agreement shown was sufficient to give general support to the estimates based on physical maximization.
Further analyses enabled maximum rainfalls for other durations to be related to the 2 and 24-hour values. For intermediate durations maximum rainfall has a straight line relationship with the log of duration and for other durations depends on average annual rainfall as shown in Table 3; for durations less than 2 hours R_{max} is related to 2 hour R_{max} while for durations 48 hours to 25 days it is related to 24 hour R_{max}.

Duration, D	**Area A (km^2)**									
	1	**5**	**10**	**30**	**100**	**300**	**1000**	**3000**	**10000**	**30000**
1 min	0.76	0.61	0.52	0.4	0.27	-	-	-	-	-
2 min	0.84	0.72	0.65	0.53	0.39	-	-	-	-	-
5 min	0.9	0.82	0.76	0.63	0.51	0.38	-	-	-	-
10 min	0.93	0.87	0.83	0.73	0.59	0.47	0.32	-	-	-
15 min	0.94	0.89	0.83	0.77	0.64	0.53	0.39	0.29	-	-
30 min	0.93	0.91	0.89	0.82	0.72	0.62	0.51	0.41	0.31	-
60 min	0.96	0.93	0.91	0.86	0.79	0.71	0.62	0.53	0.44	0.35
2h	0.97	0.95	0.93	0.9	0.84	0.79	0.73	0.65	0.55	0.47
3h	0.97	0.96	0.94	0.91	0.87	0.83	0.78	0.71	0.62	0.54
6h	0.98	0.97	0.96	0.93	0.9	0.87	0.83	0.79	0.73	0.67
24 h	0.99	0.98	0.97	0.96	0.94	0.92	0.89	0.86	0.83	0.8
48 h	-	0.99	0.98	0.97	0.96	0.94	0.91	0.88	0.86	0.82
96 h	-	-	0.99	0.98	0.97	0.96	0.93	0.91	0.88	0.85
192 h	-	-	-	0.99	0.98	0.97	0.95	0.92	0.9	0.87
25 days	-	-	-	-	0.99	0.98	0.97	0.95	0.93	0.91

Table 2. Relation of areal reduction factor (ARF) with duration (D) and area (A).

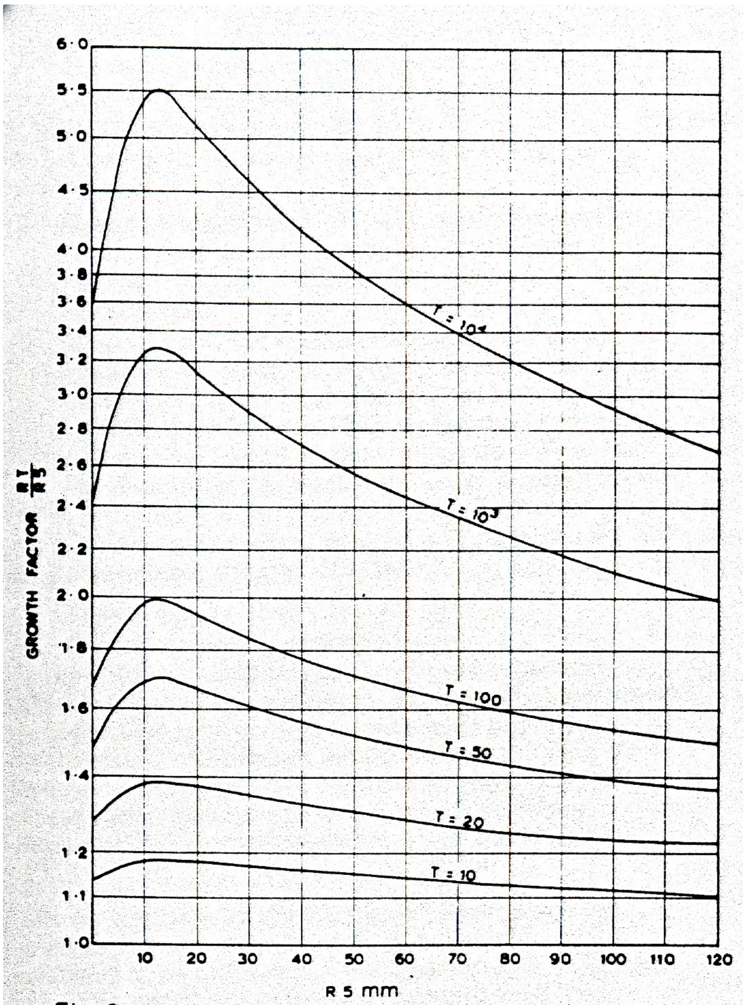


Fig. 2

GROWTH FACTOR AS A FUNCTION OF R5

3. Flood Estimation From Catchment Characteristics*

In estimating the flood of given return period, Q_T, data is required at the site, whichever of the two methods referred to in the introduction is used. In the absence of data, it is necessary to express in mathematical form the dependence of the required estimate on the physical and climatic characteristics of the catchment. By the statistical analysis approach the flood estimate is given as Q_{Bar} x (Q_T/Q_{Bar}). Q_T/Q_{Bar} is a growth factor obtained by regional pooling of flood data, as described later in Section 4.4. It can be regarded as a known constant relating Q_{Bar} and Q_T. Q_{Bar} is the mean of the annual maximum floods and exceeds the 1-year flood by about 15%; its value is given by a formula, for use in the absence of data, obtained by regression analysis of Q_{Bar} on numerically expressed catchment characteristics.

In the unit hydrograph approach, Q_T is estimated using the 1-hour unit hydrograph at the site. Its time-to peak, Tp, which was also expressed in terms of catchment characteristics for use in the absence of data, characterizes this.

* Volume 1, chapter 4

3.1 Catchment characteristics

Many measures of catchment characteristics such as area, shape, slope and length have been proposed in the past [6] but there was not general agreement on which measure should be used for each characteristic or even on which characteristics should be included in a formula. A complete examination of this problem was made in the Flood Studies and characteristics were chosen for use according to hydrological, statistical and practical criteria. The characteristics should be hydrologically relevant so that results based on them could be applied on new catchments. They should make a statistically significant contribution to the formulae and from the practical point of view should be capable of measurement from easily available material. Finally, the same characteristics should be applicable for all purposes and be of use for example in the formulae for Q_{Bar} and T_p.

3.1.1 Physiographic characteristics

The following were the measures adopted in the analyses:

(i)Size: Area (A) was the variable used; Stream Length (L) is a correlated measure,

(ii)Slope:(S) was taken as that of the main channel between two points, 10%and 85% of the stream length from the gauge.

(iii)Channel network: A stream frequency (Fs) was adopted, taken as the number of channel junctions on 1/25,000 maps divided by the basin area (km2). For Ireland junctions are counted on 1/63,360 (1-inch) maps and the result F's is converted to stream frequency by the relationship Fs = 2.8 F's - 0.172, derived by a comparison of results from the two map series for British catchments.

(iv)Storage: The index used is W = 1+ the fraction of the catchment draining through lakes as derived from 1/250,000 maps. (A lake is ignored if less than 1% of the area contributing to it). Caution is recommended, however, in using the index for catchments with large lakes as the analyses indicated that it might not fully account for their effect in reducing flood peaks.

(v)Soils and Geology. The measure adopted is based on a soil classification derived from the Soil Surveys in Great Britain and Ireland. [†] Maps (See Map 1) indicate the soil "Winter Rain acceptance potential' - which approximates to infiltration potential — ranging from a maximum for Class 1 to a minimum for Class 5. The fraction of each soil type (G1, G2, etc) is given a weighting and the soil index is the weighted mean:

$$G = \frac{0.15G1 + 0.30G2 + 0.40G3 + 0.45G4 + 0.50G5}{G1 + G2 + G3 + G4 + G5} \qquad (1)$$

Since winter rain acceptance is broadly the reverse of runoff, the relative weightings are analogous to runoff coefficients in earlier methods of flood estimation.

† The soil map for Ireland was prepared by Dr. M.J.Gardiner and Mr. L.F. Galvin of the Agricultural Institute, Dublin and by Dr. S. McConaghy Ministry of Agriculture, Belfast.

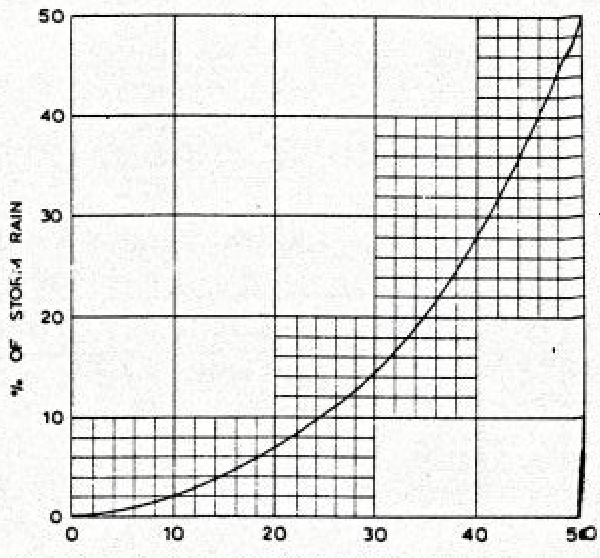


Fig. 3

THE 75% WINTER STORM PROFILE RECOMMENDED FOR DESIGN

(vi)Land Use. The Studies also attempted to examine the effect on floods of areas of forest and urban development. Neither was significant in Irish catchments but the fraction of area 'built up' (U), proved a useful variable for British catchments.

3.1.2. Climatic characteristics

To take account of local climatic affects the following two indices were used:

(i)Mean annual rainfall (R_{Bar}). The report provides maps of standard period average annual rainfall for United Kingdom (1936 — '70) and Ireland (1926-'60).

(ii)R_{smd}.* this is a measure of rainfall excess, in millimetres, given by 1-day R5 rainfall reduced by a weighted mean of annual soil moisture deficit (SMD). Map 3 shows SMD for Ireland derived from 17 meteorological stations. R_{smd} was devised to overcome the conceptual difficulty associated with R_{Bar} in that it is not directly the cause of short-term hydrological events. R_{smd} and R_{Bar} are however correlated, the approximate relation between them being $R_{smd} = 2.48\sqrt{}(R_{Bar}- 40\text{ mm.})$

*Volume 1, chapter 4.

3.2 Catchment delay parameter†

In previous investigations catchment delay parameters included time of concentration, Tc; rise time from lowest to highest stage level; time to peak from beginning of rain or from centre of mass of rainfall excess; and time from centroid of excess rain to centroid of response runoff. These parameters have been related to catchment characteristics, generally stream length and slope, frequently combined in the form $L/\sqrt{S}$.
The time to peak of the 1-hour unit hydrograph used in analyses is also a catchment delay parameter and, to enable it to be estimated in the absence of recorded data, it was related to the catchment characteristics. Thus:

$$T_P = 46.6 . S^{-0.38} . R_{smd}^{-0.4} . (1+U)^{-1.99} . L^{0.14} \qquad --- \quad (2)$$

It must be stressed that this is less reliable than that obtained from recorded data. Tp should preferably be obtained by deriving a unit hydrograph but the analyses showed that it can be estimated with reasonable reliability from the lag time of the catchment, defined as the time from centroid of total rainfall to flow peak.

The relationship is: $Tp = 0.9\ Lag$ --- (3)

The analyses also showed that an approximation of Tp, in terms of the customary $L/\sqrt{S}$, is given by

$$T_P = 2.8\left(\frac{L}{\sqrt{S}}\right)^{0.5} \qquad --- (4)$$

And that the base length of the 1-hour unit hydrograph is

$T_B = 2.52\ T_P$ --- (5)
Since $T_B - 1 = Tc$ the latter can be expressed as:

$$T_C = 7.06\left(\frac{L}{\sqrt{S}}\right)^{0.5} - 1 \qquad \text{- - - (6)}$$

In addition, its results can be compared with those of other available formulae for calculating time of concentration [(6)]

3.3 Relation between Q and catchment characteristics

Using multiple regressions, the values of Q_{Bar} from 533 stations were related to their catchment physiographic and climatic characteristics. It was found that the standard error of estimates was reduced by the inclusion of an increasing number of variables (with the exception of Urban (U) which was significant only in the Thames-Essex area where special treatment was required). The following 6 variable equation was recommended for use:

$$Q_{Bar} = C.A^{0.94}.F_s^{\ 0.27}.G^{1.23}.R_{smd}^{\ 1.03}.W^{-0.85}S^{0.16} \qquad \text{- - - (7)}$$

The constant C equals 0.0172 for Ireland but changes with geographical region in Great Britain - (the indices of the variables are the same for both countries). Similar regional equations were produced using mean annual rainfall (R_{Bar}) instead of R_{smd} and that recommended for Ireland is:

$$Q_{Bar} = 0.00042.A^{0.95}.F_s^{0.22}.G^{1.18}.R_{Bar}^{\ 1.05}.W^{-0.85}.S^{0.19} \qquad \text{- - - (8)}$$

These two equations have about the same reliability with standard factorial errors of about 1.5. This means that for two thirds of catchments the percentage error would fall between +50% and -33% and that in 1 in 20 cases they could over-estimate by more than 120% or underestimate by more than 50%. These errors were shown in the analyses to be little better than those of an estimate of Q_{Bar} based on a one-year flow record for the site. Indeed, it could be said that little extra accuracy would be lost, particularly on small catchments without significant storage, if equation 8 were rewritten as:

$$Q_{Bar} = 0.00038.A.F_s^{0.2}.G^{1.2}.R_{Bar}.S^{0.2} \qquad \text{- - - (9)}$$

(This has a standard factorial error of about 1. 8)

*Volume 1, chapter 4

† Volume 1, chapter 6, sections 2,4 and 5.

Duration D	Average Annual Rainfall, mm/100: 8 – 10	10 – 14	14 – 20	20 – 28
	Ratio $\frac{\text{max rainfall of duration D}}{\text{max rainfall of duration 2 hrs}} = \frac{D\ R_{max}}{2\ hr\ R_{max}}$			
1 min	0.06	0.06	0.06	0.06
2 min	0.11	0.11	0.11	0.11
5 min	0.23	0.23	0.22	0.22
10 min	0.36	0.36	0.34	0.34
15 min	0.47	0.47	0.45	0.45
30 min	0.65	0.65	0.62	0.62
60 min	0.83	0.83	0.79	0.79
2 hour to 24 hour	$D\ R_{max} = 2\ hr\ R_{max} + 0.93\ (24\ hr\ R_{max} - 2\ hr\ R_{max})\ \log(\frac{D}{2})$			
	Ratio $\frac{\text{max rainfall of duration D}}{\text{max rainfall of duration 24 hrs}} = \frac{D\ R_{max}}{24\ hr\ R_{max}}$			
48 hour	1.10	1.11	1.12	1.14
72 hour	1.14	1.16	1.18	1.23
96 hour	1.18	1.20	1.24	1.32
192 hour	1.28	1.35	1.49	1.62
25 days	1.68	1.92	2.20	2.49

Table 3 Maximum rainfall of duration D, D R_{max}, related to 2 hr R_{max} and 24 hr R_{max} for D = 1 min. to 25 day, and to average annual rainfall.

Equations (7), (8) and (9) should be used with caution and confined to rough flood approximations; examples are estimates required in feasibility studies or for gauging station installation and design. Improvements can be achieved by using available local information. For example, if comparison of predicted and measured values of Q_{Bar} at gauged sites in the vicinity show a consistent pattern, this can be used to improve the prediction. However, the most valuable local information is recorded flow data for the site; most projects of any magnitude allow sufficient time in the preparatory phase to collect a few years of records and these can be used to give greatly improved flood estimates. The standard errors of the above results, despite thorough research on a large amount of data, indicate that reliable flood estimates from simple methods are not attainable. The more elaborate procedures described in the succeeding sections are therefore necessary.

4. Estimation of Q_T by Statistical Methods

4.1 Flood frequency models*

In determining the relation between flood peak Q and return period, T the entire hydrograph can be replaced by the series of flood peaks alone. This can be done in two ways as follows:

(i)The partial duration series is the series of all flood peaks in excess of an arbitrary threshold, q_0. Its Q-T relation is the same as in the population of all floods and may be defined in terms of q_0, the number of peaks exceeding q_0 per year, and the distribution of peak magnitudes. It is not universally used mainly because of the difficulty in defining independence between peaks that occur close together and because it involves conditional probability arguments when it includes more than one flood per year.

The annual maximum series consists of the maximum flood peaks in each year. It is popular in flood hydrology because it obviates the difficulties associated with the partial duration series, while the relation between Q and return period T_{AM} is linked to that between Q and T in the parent series as shown in Table 4 [(9)]. This shows that T_{AM} differs from T by only 0.5 when T > 5 and means that the Q-T relation is validly obtained from the annual maximum series except for T<5.To illustrate, suppose q is exceeded 100 times in 200 years, then T = 2 years. If the exceedances are examined it will be found that less than 100, say 80, appear in the annual maximum series, because in some years second or lower ranking floods may be greater than the maxima in others. This means that among annual maxima q is exceeded less frequently than in reality and its return period is exaggerated to T_{AM} = 200/80 = 2.5 years, i.e. a decrease in frequency from 5 to 4 times a decade. When Q_T is estimated for T< 5, use of the annual maximum series is insufficient without a correction, using Table 4. Thus the 2 year flood Q_2 is the Q value corresponding to T_{AM} = 2.54 in the annual maximum series.

T in population of all floods	0.5	1	1.45	1.78	2	5	10	50	100
T_{AM} among annual maximum	1.16	1.58	2	2.33	2.54	5.52	10.5	50.5	100.5

Table 4. Relation between T and T_{AM} for fixed Q value (After Langbein [(9)])

A frequently quoted objection to the annual maximum series is that it omits some of the largest floods on record. However, Table 4 shows that for large T the annual maximum series agrees closely with the partial duration series, which includes these large floods. Neither does the latter give better estimates i.e. smaller standard errors of estimate as defined in section 4.6. The Flood studies also show that the inclusion, by lowering the threshold q_0, of extra floods in the partial duration series does not help to define better the statistical distribution of peaks. †

4.2 Statistics and distributions

Statistical analysis gives, not the Q-T relation directly but the relation between Q and probability of occurrence expressed by a distribution function F (q) = PR (Q $\leq$ q) for the series. Its complement 1— F (q) is called the exceedance probability and T = 1/(1— F (q)). The form taken by F(q) depends on the kind of series involved but it cannot be deduced by theoretical reasoning alone. Empirical studies have shown that the increase of Q with log T is either linear or slightly curved and the following recommendations are made here:

*Volume 1, chapter 2.2
†Volume 1, pp 207-209

(i)The exponential distribution**

$$PR\ (Q \leq q) = F(q) = 1 - e^{-(q-q0)/\beta}\ , \qquad \beta > 0 \qquad ---\ (10)$$

Is recommended for the partial duration series, β is the amount by which the mean exceeds the threshold q_0 and the corresponding Q-T relationship is

$$Q_T = q_0 + \beta \ln T = q_0 + \beta\ y \qquad ---\ (11)$$

Where y, the reduced exponential variate, is distributed as in (10) with $q_0 = 0$ and $\beta = 1$. This is illustrated by the linearised graph in Figure 4. The distribution is used in this form in data analysis, usually with q as ordinate and y as abscissa.

(ii)The extreme value family of distributions†† is recommended for the annual maximum series, the general extreme value distribution being

$$PR(Q \leq q) = F(q) = e^{-(1-\frac{q-u}{\alpha}k)^{1/k}}\ , \alpha > 0 \qquad ---\ (12)$$

In which the Q-T relation, in terms of y_T is:

$$Q_T = u + \alpha \left(\frac{1 - e^{-ky}T}{k} \right) \qquad ---\ (13)$$

The mathematical limit of this distribution as $k \rightarrow 0$ is called the extreme value type 1 or Cumbel distribution [(10)]

$$PR\ (Q \leq q) = F(q) = e^{-e-(q-u)/\alpha} \qquad ---\ (14)$$

And the Q-T relation is:

$$Q_T = u + \alpha\ y_T \qquad ---\ (15)$$

†† Volume 1, chapter 1.2.4
** Volume 1, chapter 1.2.3

The quantity $y_T = -\ln\left[-\ln\left(\frac{T-1}{T}\right)\right]$, ($\approx$ ln (T-1/2), when T $\geq$ 5, is called the 'Gumbel reduced variate, tabulated in Table 5. The extreme value type 1 distribution is illustrated in Figure 5, and the Q-T relation for the general extreme value distribution in Figure 6. In the latter, the straight line, corresponding to k =0, is the type 1 distribution.

T	2	5	10	20	25	50	100
Y_T in extreme value type 1 = -ln(-ln(T-1)/1)	0.37	1.5	2.25	2.97	3.2	3.9	4.6
Y_T in exponential = ln(T)	0.69	1.61	2.3	3	3.22	3.91	4.61

Table 5. Relation between Y_T and T in extreme value type 1 and exponential standardized distributions.

It was found during the Flood Studies that the extreme value distribution best suited to Ireland has a small negative k value (k= -0.05), see Figure 7. Because of the slight curvature, it could be approximated by a straight-line meaning, that the two-parameter extreme value type 1 (Gumbel) distribution is adequate in Ireland for most purposes.

4.3 Common quantities in flood series

4.3.1 Annual maximum series

Any series can be summarised, at the expense of detail, by its mean, μ and standard deviation σ . In the annual maximum series q1, q2…qi…q_N from N years of record the mean annual flood is estimated by:

$$Q_{Bar} = \frac{1}{N}\sum q_i \qquad \text{- - - - (16)}$$

(In the absence of flow data Q_{Bar} is obtained from Equation (7)).
The standard deviation is defined as:

$$\sigma = \sum (q_i - Q_{Bar})^2 /(N-1) \qquad \text{- - - - (17)}$$

And measures the scatter of the values about the mean.

The coefficient of variation is a useful dimensionless form and in Ireland has an observed average value of 0.3. (Britain is about 0.4)

$$C_V = \frac{\sigma}{\mu} \text{ or } \frac{\sigma}{Q_{Bar}} \qquad \text{- - - - (18)}$$

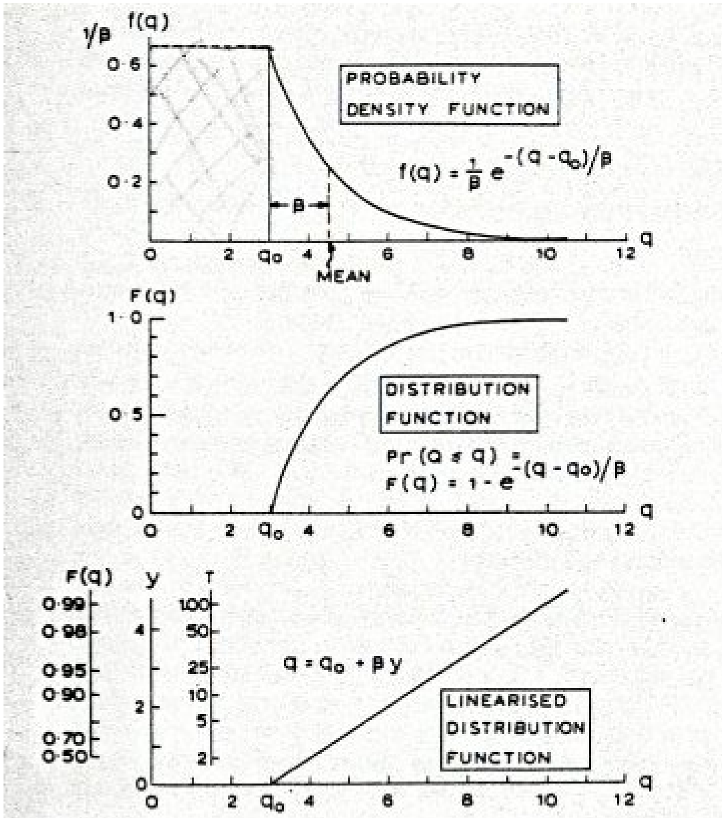


Fig. 4

THE EXPONENTIAL DISTRIBUTION

In the linearised graph scales are determined by the parameter free distribution, $F(y) = 1\text{-}e^{-y}$; $y = -\ln(1-F) = \ln T$

If Q is distributed as extreme value type 1 the following relations hold

$\mu = u + 0.577\alpha$ ---- (19a)

$\sigma = \alpha / 0.78$ ---- (19b)

And the mean annual flood has return period T = 2.33 among annual maxima, i.e. Q_{Bar} is exceeded by 10 annual maxima on average in every 23 years. The area to the right of $\mu = u + 0.577\alpha$ on Figure 5 is the exceedance probability of μ among annual maxima. Using PR $(Q > \mu) = 1 -$ PR $(Q \leq \mu) = 1 -$ — $F(\mu)$ and equation (14) this is shown to be 0.43 and its reciprocal is the return period $T_{AM} = 2.33$. Table 4 shows that T = 2.33 among annual maxima implies a T value of 1.78 in the population meaning that it is actually exceeded 10 times on average in every 18 years.

4.3.2 Partial duration series

The probability that any peak exceeds q is, by equation (10),

$$PR\,(Q > q) = e^{-(q-q0)/\beta} = 1/T^1$$

Where T^1 is the average number of peaks occurring in the series between successive exceedances of q.

4

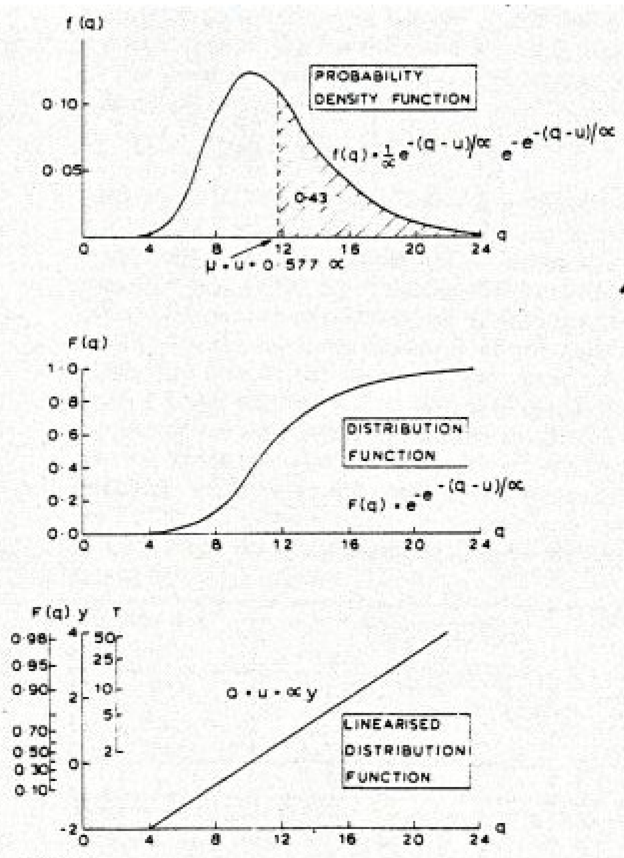


Fig 5

THE EXTREME VALUE TYPE 1 OR GUMBEL DISTRIBUTION

In the linearised graph scales are determined by the parameter free distribution

$$F(y) = e^{-e^{-y}}; \quad y = -\ln - \ln F = -\ln\left[-\ln\left(\frac{T-1}{T}\right)\right].$$

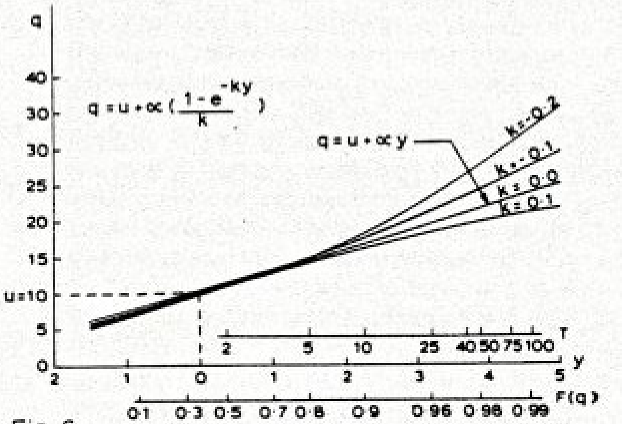


Fig. 6

MAGNITUDE RETURN PERIOD RELATION IN GENERAL EXTREME VALUE DISTRIBUTIONS

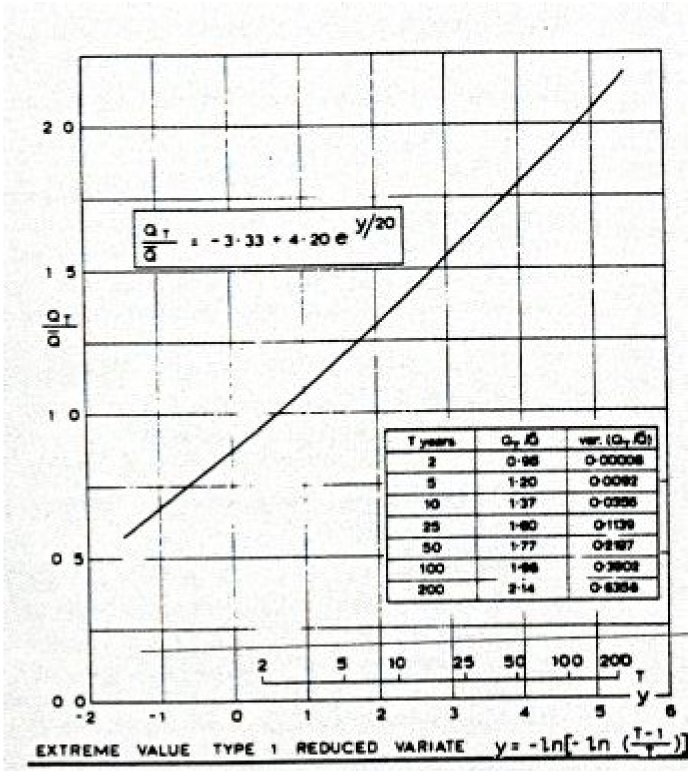


Fig. 7

AVERAGE $\frac{Q_T}{\bar{Q}}$ - T RELATION FOR IRELAND

Hence $T^1 = e^{(q-q0)/\beta}$ or $q = q_0 + \beta \ln T^1$

If λ is the average number of peaks per year exceeding q_0 then T^1 peaks occur in $T = T^1/\lambda$ years. Therefore the relation between Q_T and T in years is:

$$Q_T = q_0 + \beta \ln \lambda T$$

$$=(q_0 + \beta \ln \lambda) + \beta \ln T \qquad \text{---- (20)}$$

In particular, the one-year flood, T = 1,is

$$Q_1 = q_0 + \beta \ln \lambda \qquad \text{---- (21)}$$

This magnitude has return period T = 1.58 in the annual maximum series, see Table 4, and this corresponds to:

$$y_T = -\ln\left[-\ln\left(\frac{(T-1)}{T}\right)\right] = -0.002 \approx 0$$

In the extreme value reduced variate. The value of Q_T/Q_{Bar} in the average distribution of Q_T/Q for Ireland (Figure 7, section 4.4) for this value of y is 0.87. Therefore:

$$Q_1/Q_{Bar} = 0.87$$

Or $\quad Q_{Bar} = 1.15Q_1 \qquad$ - - - - (22)

Thus the mean, Q_{Bar} of the annual series is 15% larger than the one-year flood.

3.3 Statistical estimation

Apart from histograms which are useful only for display of data, methods used for examination of a series and for estimating the Q-T relationship are as follows: The probability plot comprises a graph corresponding to the linearised portion of Figures 4 and 5 prepared from the sample data. Histogram data cannot be plotted directly on it because the linearisation introduces the reduced variate y, linearly related to Q and is such that either $F = 0$ is represented by $y = -\infty$ or $F = 1$ by $y = \infty$. Instead, the ranked flood values are paired with values of plotting positions, y, which may either be calculated or read from appropriate tables, as explained in Section 4.5.3. Q is then plotted against y on ordinary graph paper and a return period scale marked alongside the y-axis. Alternatively, probability paper, in which the abscissa is graduated in terms of probability, may be used with the plotting position specified as probability or F values, viz:

$$F_i = \frac{i - 0.44}{N + 0.12}$$

For Gumbel paper, where i = 1 refers to the smallest value. In graphical estimation, an eye guided smooth curve or line is drawn to give Q values of any specified return period. The major criticism of this method is the differences between curves produced by different analysts. In numerical estimation, a form of distribution function, F (Q), is adopted using experience or a probability plot as a guide. This function contains parameters which are unknown but which can be expressed in terms of observed data by analytical rules known collectively as estimators. Different estimator types exist depending on the criterion adopted to express agreement between distribution and observed data. The moments and least squares estimators are discussed in Section 4.5.3. A difficulty arises when selecting the form of F (Q) and the choice involves some element of subjectivitiy, which should be borne in mind when criticizing the graphical method. If the correct choice is made then numerical estimation is superior. Furthermore, a measure of efficiency can be associated with the method chosen, in terms of the proportion, incorporated in an estimate, of all available information in the data about the distribution.

4.4 The Regional Curve approach*

If there are several records from the same flood population an-estimate of Q_T can be obtained from each. The standard deviation of these estimates is called the standard error of estimate (se) of Q_T and about two thirds of the estimates fall in $Q_T \pm$ se (Q_T). Table 6 shows se (Q_T) as a percentage of Q_T for different N and T values in the extreme value type 1 distribution. It is given approximately by:

$$\mathrm{se}\,(Q_T) = \frac{0.4Q_{Bar}}{\sqrt{N}}(0.35 + 0.80\,y_T) = ExQ_{Bar} / \sqrt{N} \qquad \text{- - - - (23)}$$

With y-p as in Table 5, row 2 and E as tabulated in the last row of Table 6. These are minimum figures based on maximum likelihood estimation. In practice, the true distribution will differ from the assumed one in which case the true standard error could be double the tabulated values. These very large unavoidable standard errors mean that rare floods cannot be estimated with reliability from a single station record. Another disadvantage of the single station estimate is the outlier problem. This occurs when a series contains one extremely large flood, which does not conform to the pattern of the remaining points on a probability plot, and it is necessary to decide how much weight it should be given in determining the Q-T relation.

	Return Period, T				
Sample Size N	**2**	**10**	**25**	**100**	**1000**
10	12.3	15.01	16.29	17.72	19.28
25	7.84	9.5	10.3	11.2	19.2
50	5.54	6.71	7.29	7.9	8.62
100	3.92	4.75	5.15	5.6	6.1
E	0.26	0.86	1.16	1.61	2.35

Table 6. Standard error of Q_T estimated as % of Q_T.
Extreme value type 1 distribution, 0.38u, C_V = 40%

To overcome these difficulties the regional curve method examines annual maximum series at several stations jointly. The series are rendered homogeneous by dividing each by its mean Q_{Bar}, and the relation between Q_T/Q_{Bar} and T is then estimated, using a modified form of Dalrymple's technique of regional pooling of data [(11)]. The relation is effectively the mean pattern established by the individual probability plots viewed concurrently, and the curve is extended by exploiting the statistical independence of stations, geographically well spaced. If 10 such stations have 200 station years of data between them the four largest values of Q/Q_{Bar} among them are plotted as the four largest in a sample of 200. This is repeated for other groups of independent stations and the curve is drawn through the mean of all these points. Ireland was treated as one region and the resulting Q-T relation is shown in Fig. 7[†]. It takes the form of a general extreme value distribution with parameters u = 0.87, α = 0.21, k = -0.05.
To apply this curve, the mean annual flood Q_{Bar} is estimated from a record of flows or in their absence from the catchment characteristics using equation (7); then Q_{Bar} is

multiplied by the ordinate Q_T/Q_{Bar} corresponding to T in Figure 7. In this method the standard error of estimate is:

$$se(Q_T) = \sqrt{Q_{Bar}{}^2 . var(Q_T / Q_{Bar}) + (Q_T / Q_{Bar})^2 \, var(Q_{Bar})} \qquad ---- (24)$$

*, † Volume I, chapter 2.6.

Where (Q_T/Q_{Bar}) is the ordinates from figure 7 and var (Q_T/Q_{Bar}) is tabulated there.

Var $Q_{Bar} = 0.16\, Q_{Bar}{}^2$ if Q_{Bar} is estimated from catchment characteristics.

$$= \frac{0.16\, Q_{Bar}{}^2}{N} \text{ if } Q_{Bar} \text{ is estimated form N years data} \qquad ---- (25)$$

4.5 Examples*

Estimates of the 25-year flood, Q_{25}, are obtained below for the Owengariff River at Torc Weir, Co. Kerry. The catchment's characteristics are listed in Table 7. A 28-year record is available but estimates also assume a shorter record and no data at the site.

4.5.1 No Records

The two quantities Q_{Bar} (mean annual flood) and Q_{25}/Q_{Bar} are used in this case. The latter is obtained as 1.60 from Figure 7 while Q_{Bar} is obtained from equation 7

$Q_{Bar} = (0.0172)\,(8^{0.94})\,(1.93^{0.27})\,(0.45^{1.23})\,(74.7^{1.03})\,(74.5^{0.16}) = 9.21$ cumec.

Hence, if $Q_{25} = Q_{Bar} \times (Q_{25}/Q_{Bar}) = 1.60\, Q_{Bar} = 14.7$ cumec.

For the standard error calculation, note that var $Q_{Bar} = 0.16\, Q_{Bar}{}^2 = 13.57$ from equation (25) and var $Q/Q_{Bar} = 0.1139$ from Fig. 7. Then equation (24) gives se (Q_{25})= 6.7 cumec = 45% of Q_{25}. The ability of equation (7) to estimate Q_{Bar} should be checked on neighboring catchments for which records are available. Consistent under or over-estimation on these catchments should be taken into account by applying a percentage correction at the site. †

A =	8 km²	R_{bar} =	2335 mm
L =	3.04 km	2 day R5 =	113 mm
S =	74.5 m/km	R_{smd} =	74.7 mm
F_s =	1.93 j/km²		
G =	0.45		

Table 7. Catchment characteristics for River Owengariff at Torc Weir.

* Volume 1,chapter 2.11
† Volume 5, Figure 1.4.23

4.5.2 Five Years Records (1942-47)

When a short record is available, it is used to give an estimate of Q_{Bar} either from the annual maxima or the partial duration series. The latter method is not affected, by a single large flood in a short record, to the extent that the annual maximum method is. The factor Q_{25}/Q_{Bar} = 1.60 is again multiplied by the resulting Q_{Bar} value. The 15 largest floods in the 5 years 1942 — 1947 are listed in Table 8 with the annual maxima asterisked. The calculations are 'set out in the table and it is seen that Q_{Bar} is just over 6 cumecs as compared with 9.21 by equation 7. The figure from the data is always preferable. Q_{25} is now estimated as ___ cumecs by annual maxima and as 10.02 by the partial duration series and se (Q_{25}) is reduced to 27%.

4.5.3 More than 10 years record

When 10 or more years data are available at the site it is usual to inspect the peaks on a probability plot. This can be done on an annual maximum or partial duration series and both are illustrated here. The Q-T relation obtained from the data by either method will almost certainly be different from that obtained from the regional curve, Fig. 7. This must be expected because randomly drawn records from the countrywide population display a large scatter among themselves.

Year 1942-43	1943-44	1944-45	1945-46	1946-47
5.81*	6.09*	6.09*	5.02* (This is not among largest 15)	7.89*
5.54	6.09	5.54		6.09
5.54		5.54		5.54
5.28		5.54		
5.17		5.28		

Annual Maximum series $\bar{Q} = (5.81 + 6.09 + 6.09 + 5.02 + 7.89)/5 = 6.18$

$Q_{(25)} = 1.60\,\bar{Q} = 9.89$ cumec.

Equation (25) gives var $\bar{Q} = 0.16\,\bar{Q}^2/5 = 1.22$ and

var $Q_T/\bar{Q} = 0.1139$ as above

Therefore equation (24) gives se $(Q_{25}) = 2.73 = 27\%$ of Q_{25}.

Partial duration series	No. of peaks	Mean of peaks	Minimum peak	Peaks per year
	$M = 15,$	$\bar{q} = 5.80,$	$q_{min} = 5.17,$	$\lambda = M/5 = 3$

$$\beta = \frac{M(\bar{q} - q_{min})}{M-1} = 0.675, \quad q_0 = q_{min} - \beta/M = 5.125$$

$$\bar{Q} = q_0 + \beta \ln\lambda + 0.577\beta = 6.26, \quad Q_{25} = 1.60\,\bar{Q} = 10.02$$

Table 8. Q_{25} estimated from 5 years record. (Expressions for β, q_0 and $\bar{Q}$ from Vol. 1 Chapter 2, eqns. 2.7.5.4, 2.7.5.5 and 2.7.9.1).

The analysis is illustrated by the use of the 28 years record, which is long by normal Hydrological standards.
It is recommended that a distribution fitted to a record of length N should not normally be used to estimate Q_T for T>2 N. For large T the relation Fig. 7 should be used in conjunction with Q_{Bar}.

(i)Annual maximum series

The extreme value type 1 distribution is fitted to the 28 annual maxima in Table 9. The plotting positions, y, given in column 4 are taken from Table 1.1.16 of the Flood Studies Report. (Columns 8 and 9 show how they may be approximated). The ranked flood values are shown plotted against them on Figure 8, on which a return period scale is marked using Table 5.
The least squares estimates of the parameters u and are obtained by regression of the ranked flood values on the plotting positions, y. The equations used are those of ordinary regression analysis.

$$\alpha = \frac{\sum (q_i - Q_{Bar})(y_i - y_{Bar})}{\sum (y_i - y_{Bar})^2} \quad \text{And} \quad u = Q_{Bar} - \alpha\, y_{Bar}$$

And are evaluated in Table 9 using the identities

$$\sum (q_i - Q_{Bar})(y_i - y_{Bar}) = \sum q_i y_i - NQ_{Bar} y_{Bar} \qquad \text{And}$$

$$\sum (y_i - y_{Bar})^2 = \sum y^2{}_i - Ny_{Bar}{}^2$$

Giving u = 5.26 and α =1.40 as shown. The Q-T relation is therefore $Q_T = u + \alpha\, y_T$ = 5.26 + 1.40 y_T which is shown in Fig. 8 where y_T is as given in row 2 of Table 5 (y_T = 3.20 for T = 25). Therefore Q_{25} = 5.26 + 1.40 x 3.20 = 9.74 cumec.
Equation (23), with E = 1.16 from Table 6 and Q_{Bar} = 6.07 from Table 9, gives se (Q_{25}) = E. Q_{Bar} / √28 = 1.33 = 14% of Q25.
The least squares method is recommended but a shorter method is to approximate the distribution moments by the sample moments and to obtain u and α from them using equations 19 (a) and 19 (b) as shown in Table 9.

Annual Maxima q_i	Rank	Ranked Annual Maxima $q_{(i)}$	Plotting Position	q_i^2	v_i^2	$q_{(i)}v_i$	F_i = I - 0.44/N+0.12	y_i = -ln-lnF_i
5.81	1	4.15	-1.32	17.22	1.74	-5.48	0.0199	-1.37
6.09	2	4.24	-1.04	17.98	1.08	-4.41	0.0555	-1.06
6.09	3	4.53	-0.86	20.52	0.74	-3.9	0.091	-0.87
5.02	4	4.53	-0.71	20.52	0.5	-3.22	0.1266	-0.73
-	-	-	-	-		-	-	-
-	-	-	-	-		-	-	-
-	-	-	-	-		-	-	-
-	-	-	-	-		-	-	-
9.22	25	7.89	2.02	62.25	4.08	15.94	0.8734	2
4.53	26	9.22	2.37	85.01	5.62	21.85	0.909	2.35
4.53	27	9.92	2.89	98.41	8.35	28.67	0.9445	2.86
4.24	28	11.41	3.91	130.19	15.29	44.61	0.98	3.91
170.07	N = 28		16.19	1117.6	50.28	155.68		

Method Of Moments:

$$Q_{Bar} = \sum \frac{q_i}{N}, \mathrm{Var\,Q} = \left(\sum q_i^2 - NQ_{Bar}^2\right)/(N-1), \sigma = \sqrt{\mathrm{Var\,Q}}, \alpha = 0.78\sigma, u = Q_{Bar} - 0.577\alpha$$

=6.07 =3.18 =1.78 =1.39 =5.27

$$Q_T = u + \alpha\ v_T = 5.27 + 1.39\,v_T$$

Method Of Least Squares:

$$y_{Bar} = \sum \frac{y_i}{N}, SSY = \sum y_i^2 - Ny_{Bar}^2, SQY = \sum q_i y_i - NQ_{Bar}y_{Bar}, \alpha = \frac{SQY}{SSY}, u = Q_{Bar} - \alpha y_{Bar}$$

=0.58 =40.86 =57.1 =1.4 =5.26

$$Q_T = u + \alpha\ v_T = 5.26 + 1.40\,v_T$$

Table 9. Annual Maximum Series Calculation

(ii)Partial duration series

For the 28 years record, Table 10 shows the highest 84 floods, the 3 per year series. Assuming the exponential distribution these are shown plotted on a simple exponential base y on Figure 9, with a return period scale (T^1) marked with the help of Table 5, row 3. The plotting positions are y1 = 1/84 for the smallest value, y2 = 1/84 + 1/83, y3 = 1/84 + 1/83 + 1/82 and so on as shown.

For numerical estimation the mean q_{Bar} and minimum q_{min} are noted from the data and from these, the parameters β = 0.92 and q_0 = 4.76 are computed. The third parameter λ has been fixed at 3 (floods per year). Using equation (20) the Q-T relation is

$Q_T = q_0 + \beta \ln 3 + \beta$

$\ln T = 4.76 + 0.92 \ln 3 + 0.92 \ln T = 5.77 + 0.92 \ln T.$

Therefore Q_{25} = 8.73 cumecs.

On Figure 9, the line $Q = q_0 + \beta \ln T^1$ is drawn to represent the estimated population line. (Note that T^1 is not years, but the number of floods occurring on average in the series between exceedances of Q). It can be seen that the lower points dominate the slope of the line and that it underestimates the upper values. A better fit to the data would be obtained by a curve (e.g. an eye guided one). This shows that the inclusion of the lower values offer little or no help in defining the upper part of the curve unless the same curve is followed by both high and low values and its form is known. Since the latter is unknown, it may be better to discard the low values and work with the one per year or annual exceedance series.

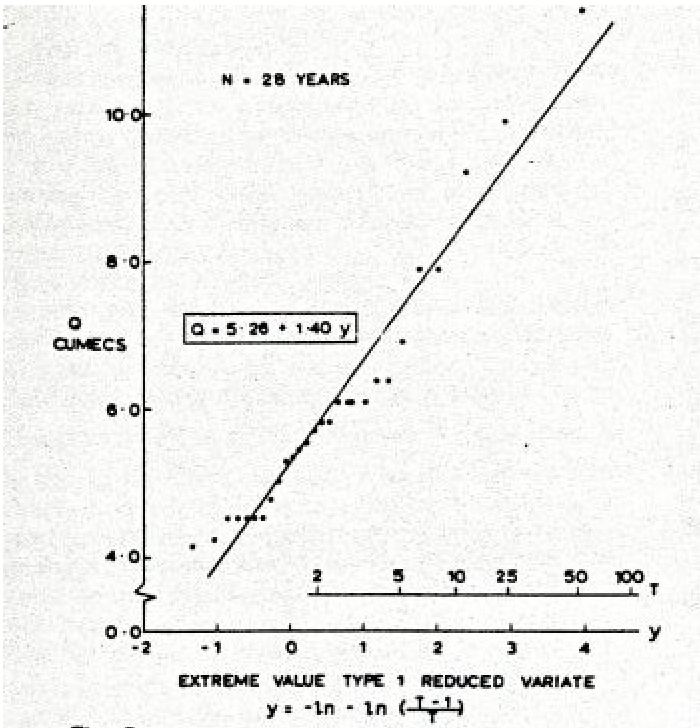


Fig. 8

PROBABILITY PLOT OF ANNUAL MAXIMUM FLOODS RIVER OWENGARIFF AT TORC WEIR

This latter series is illustrated in figure 9 and calculations similar to those shown in table 10 yield $q_{Bar} = 6.76$, $q_{min} = 5.54$, $M = 28$. Hence $\beta = 1.27$ and $q_0 = 5.49$ giving $Q_T = 5.49 + 1.27 \ln T$.
Therefore $Q_{25} = 9.57$ cumec.

4.5.4 Choice of method

In all methods, the flood estimate changes with the amount of local data used but as the latter increases the standard error decreases. This is to be expected because of the statistical nature of the material. The published statements for and against both the annual maximum and partial duration series methods are many but this paper allows them equal standing. It is usually a matter of personal choice and convenience, the annual maximum method being the less time consuming.

Rank	Peak Values $q_{(i)}$	Plotting Position y_i	Rank i	Peak Values $q_{(i)}$	Plotting Position y_i
1	4.77	0.012	.	.	.
2	4.77	0.024	.	.	.
3	4.77	0.036	.	.	.
4	4.77	0.048	77	7.26	2.421
5	4.77	0.061	78	7.26	2.564
6	4.77	0.074	79	7.89	2.731
7	4.82	0.086	80	7.89	2.931
8	4.87	0.099	81	7.89	3.181
.	.	.	82	9.22	3.514
.	.	.	83	9.92	4.014
.	.	.	84	11.41	5.014

q_{min} = minimum = 4.77 $\bar{q}$ = average = 5.68

M = No. of peaks = 84

$$\beta = \frac{M(\bar{q} - q_{min})}{M-1} = 0.92, \quad q_0 = q_{min} - \frac{\beta}{M} = 4.76$$

Table 10. Scheme of partial duration series calculations, (The expressions for and q_0 are given on page 194, Volume 1).

Estimation of T Year Hydrograph by Unit Hydrograph.

5.1 Introduction

The use of rainfall to estimate floods has a long history, beginning with the Rational Method. However, this implies a rectangular impulse response for the catchment, which means that the result obtained is extremely sensitive to changes in the time of concentration, a quantity which cannot be obtained with reliability for large catchments (ref. 6). This disadvantage is overcome if a single peaked impulse response having a more natural shape is used, because it allows a balance to be achieved between increasing storm duration and decreasing intensity while preserving the peak magnitude. This can be achieved with the unit hydrograph.
By definition the Rmm τ -hour unit hydrograph (τ UH) is the runoff hydrograph of quick response to Rmm of net (or effective) rainfall generated uniformly over the catchment area at a uniform rate for τ hours. By definition also, any change in the volume of rain in the same τ hours produces a proportional change in the runoff and the total runoff from rainfall occurring over a succession of τ hour durations can be obtained by lagging and addition. If the net rainfall in successive intervals is r1, r2 ... r1; the UH ordinates at the same intervals h1, h2, h3 and the runoff q1f, q2…q_i: then the q's can be expressed in terms of the r's and h's by the so called convolution equations

q1 = r1 h1
q2 = r2 h1 + r1 h2 - - - - (26)
q3 = r3 h1 + r2 h2 + r1 h3
q4 = r4 h1 + r3 h2 + r2 h3 + r1 h4

The unit hydrograph provides a flood hydrograph, and this is important for flood problems involving volumes or flood attenuation by storage; the UH is therefore complementary to statistical peak estimation, discussed in section 4. It is also more sensitive to the flood producing capacity of catchments than simpler techniques such as the Rational Method because it comprehends more of the factors affecting runoff. The recommended method takes account of the wetness of the catchment and its characteristics including soil cover and land use; the storm magnitude, and its duration and time distribution for the required return period, are provided for.

The method is time consuming and involves several steps* relating to the processes involved in determining the unit hydrograph, the rainfall input, the % runoff, and computing the hydrograph. These processes can be modified to give an estimate of the maximum flood, including a contribution by snowmelt (Section 6). Application of the unit hydrograph method is illustrated here by a worked example; the catchment used is again the Owengarriff River at Tore Weir whose characteristics are given in Table 7.

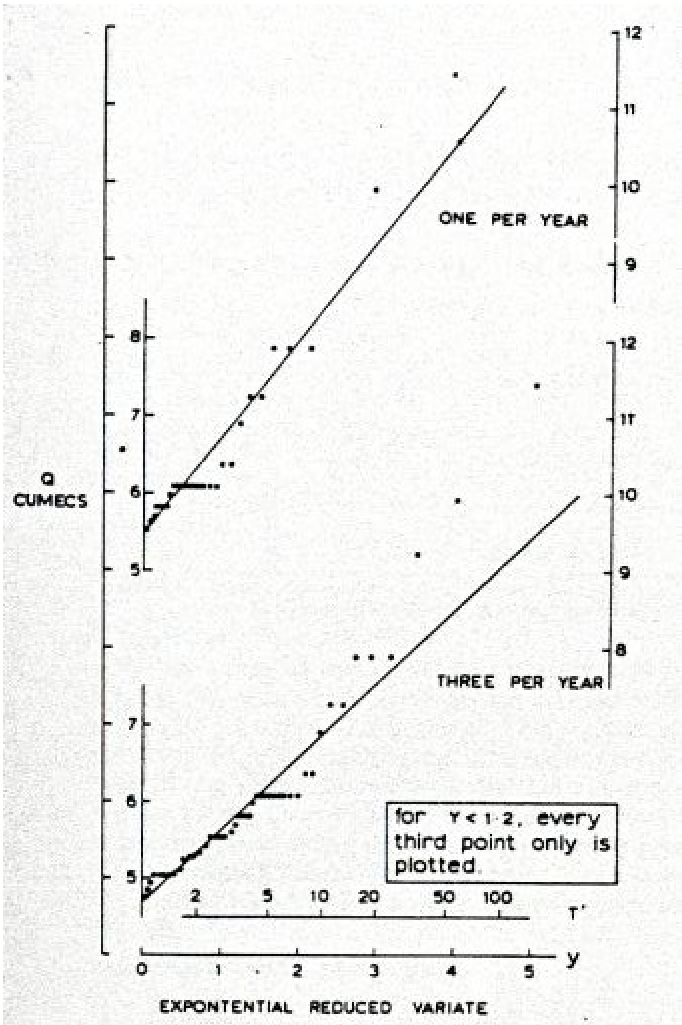


Fig. 9 y = 1n T'

PARTIAL DURATION SERIES FOR OWENGARIFF AT TORC WEIR.

5.2 The unit hydrograph†

The Flood Studies showed that a unit hydrograph is adequately represented by a triangle in the design case and that it can be described by one parameter T_p, the time to peak. For the 10 mm one hour unit hydrograph, representing the flood runoff from 10 mm of net rain occurring uniformly during one hour and over the catchment area, the peak flow $Q_P = 220/T_P$ cumec/100km^2 and base width $T_B = 2.52\ T_P$ hours.
The quantity T_P can be obtained as discussed in Section 3.2 depending on the data available at the site. Here we assume that no records are available and using equation (2), and the catchment characteristics in Table 7,

$$Tp = (46.6)\ 74.5^{-0.38}\ 74.7^{-0.4} 1^{-1.99} 3.04^{0.14} = 1.88 \text{ hours.}$$

As in the case of Q_{Bar}, the result given by this formula should be checked on neighboring catchments for which rainfall and runoff records exist and any necessary adjustment should be made. Where rainfall and flow data are available, an improved value of T_P is given by equation 3 (but in such circumstances the data should preferably be used to derive the unit hydrograph). The data interval τ for hydrograph and rainfall tabulations should be taken as $T_P/5$ approximately; $\tau = 0.4$ hours is used. The unit hydrograph specified by Q_P, T_P and T_B above is for a data interval of $\tau = 1$ hour and for different values of τ the UH has to be altered. The normal method is to use the S-curve but a convenient approximation is to alter T_P, thus

$$T_P^{\,1} = \text{New } T_P + \left(\frac{\tau - 1}{2}\right) = 1.58, \text{ say } 1.6 \text{ hours}$$

Hence $Q^1_P = 220/T^1_P$ cumec/100 km^2 = 137.5 cumec/ 100 km^2 = 11.0 cumec and $T_B = 2.52\ T^1_P = 4.03$ hours.

5.2.1 Unit hydrograph ordinates

The triangular hydrograph obtained has now to be expressed as ordinates at 0.4 intervals for the convolution with rainfall. Let the slopes of the rising and falling limbs of the hydrograph be s1 and s2.

Then $s1 = Q_P/T^1_P = 11.00/1.6 = 6.88$ and $s2 = Q_P/ (T_B - T^1_P) = 11.00 / (4.03 - 1.60) = 4.53$.

The ith ordinate hi depends on whether it is to be left or right of the peak, i.e. on whether $i.\ \tau < T^1_P$ or $> T^1_P$.

Therefore $hi = i\ s1$ if $i\ \tau < T^1_P$.

$= (T_B - i\tau)\ s2$ if $T^1_P < i\tau < T_B$

The ordinates are tabulated in Table 11; h1 to h3 use s1 and the remainder s2.

i =	1	2	3	4	5	6	7	8	9	10
t = it	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4
hi =	2.75	5.5	8.23	11.01	9.2	7.38	5.57	3.76	1.95	0.14

Table 11. Calculation of UH ordinates, $h_1, h_2, \ldots h_i, \ldots$

5.3 The design storm**

Having established the unit hydrograph ordinates, the design rainstorm must then be determined. This involves the storm return period, its duration and profile within that duration and the percentage runoff. The designer must specify the flood return period, in consultation with his clients. He also has some scope for exercising his judgment in the choice of storm profile, soil moisture deficit and antecedent precipitation index used in determining percentage runoff. However, standard values of these are recommended for use in design and these should not be departed from without good reason. (The exception to this is estimating the maximum flood -Section 6).

5.3.1 Design storm return period, duration and amount

It cannot be assumed that a T year storm produces a T year flood and the matter is further complicated because storms of different durations may cause a T year flood. Extensive work carried out during the Flood Studies has provided a means of selecting a storm duration and return period to be used with the UH to give the magnitude of the T year flood.
The investigations showed that the storm duration should be taken as

$D = (1.0+ R_{Bar}/1000)\ T^{1}_{P}$......................(27)

In the example D = (1 + 2.335) 1.60 = 5.34, use 5.2 hours. The resulting flood is not sensitive to a small change in the value of D, unlike the Rational Method, so a convenient value may be adopted without affecting the outcome. This point is important because of the prominent historical position held by storm duration in flood design and because of its critical effect on results given by the Rational Method.
The relationship between flood return period and storm return period is given in Table 12. For Q_T where T = 25 the storm return period is 42 and the corresponding rainfall depth for duration 5.2 hours is determined by the methods described in Section 2, from Vol. II of the Report.

(i) 2-day R5 is found as 113 mm from Map 2.
(iii) r = 60min R5/2 day R5 is found as 0.17 from Map 3.

* Volume 1, chapter 6.1.4, 6.8.2
** Volume 1, chapter 6, sections 7 and 8.
† Volume 1, chapter 6, sections 2,5 and 8.4

(iii)r = 0.17 and D = 5.2 hours implies r_D = 0.36 from Figure 1.
(iv)5.2 hour R5 = r_D x 2-Day R5 = 0.36 x 113 = 40.7 mm.
(v)Growth factor R42/R5, for R5 = 41, is 1.49 from Fig. 2
(vi)5.2-hour R42 = 1.49 x 40.7 = 60.6 mm.
(vii)A = 8 km^2, D = 5.2 hours give areal reduction factor = 0.96 from Table 2.
(viii)Design storm areal rainfall over catchment R = 0.96 x 60.6 = 58.2 mm.

Return Period of required flood	2.33	5	10	20	25	50	100	250	500	1000
Storm return Period	2	8	17	35	42	80	140	300	250	1000

Table 12. Storm return period to give flood of stated return period when used with prescribed rules for duration, profile and % run off.

5.3.2 Percentage runoff*

The Flood Studies concluded that in storm estimation the net response of the catchment is satisfactorily represented by a percentage runoff. This consists of two components, a fixed one depending on surface cover (soil, G and urban, U) and a changeable one depending on rainfall depth and a catchment wetness index (CWI) based on soil moisture deficit and antecedent precipitation during the previous five days. Recommended CWI values for standard design conditions are shown in Fig. 10. Percentage runoff is given by

Pr = 95.5 G + 12U + 0.22 (CWI-125) +0.1 (R-10)------(28)

In the example

Pr = 95.5 (0.15) + 0 + 0.22 (2) + 0.1 (48.2) = 48.2%

Therefore net rainfall = R x Pr = 58.2 x 0.482 = 28.1 mm. On impermeable soils, G close to 0.5, the first term dominates the expression for Pr but with highly permeable soils on gentle slopes, G close to 0.15, the last two terms contribute significantly.

5.3.3 Storm profile

The storm profile is discussed in Section 2.3. for standard design purposes on rural catchments the 75% winter profile is recommended (Figure 3). This is a symmetrical profile with a central peak and is such that only 25% of winter storms are peakier than it. The net rainfall = 28.1 mm occurring over D = 5.2 hours is distributed over 13 time intervals of τ =0.4 each. Calculations of the profile are shown in Table 13. Each interval occupies 7.69%(= 100/13) of the storm duration and because of symmetry, the last six intervals need not be tabulated. The final row is entered in cm. because the UH is the response to 1 cm. of rain.

*Volume 1, chapter 6, sections 4,5 and 7.

5.4 The Flood hydrograph

Let q1, q2, q3... be the ordinates of the storm runoff, r1, r2…be the net rainfall ordinates (Table 13) and h1, h2, h3...be the UH ordinates (Table 11). Then q1, q2, q3…are obtained by the convolution equations (26).

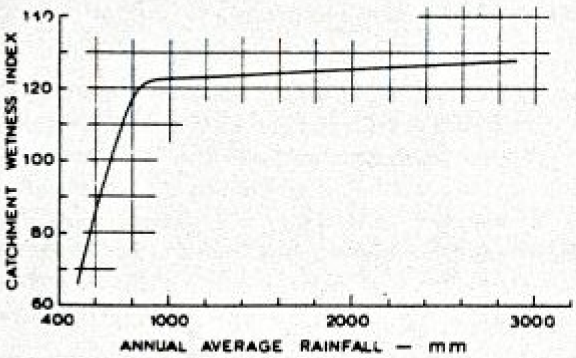


Fig. 10

DESIGN CATCHMENT WETNESS INDEX

Interval of 0.4 hr.	1	2	3	4	5	6	7
% of storm duration	7.69	15.38	23.08	30.77	38.46	46.15	53.85
% of storm rain (Fig. 3)	1.50	4.25	9.00	15.00	25.50	40.25	59.75 = 100-40.25
% of storm in interval	1.50	2.75	4.75	6.0	10.50	14.75	19.50
Net rain in interval, cm.	.042	.077	.133	.169	.295	.414	.548

Table 13 Distribution of net rainfall according to 75% winter profile.

This calculation can be set out in tabular form, where the h values are listed along the top row and the r-values along the left-hand column. In column 1, h1 is multiplied by ri and entered in row i for each i. In column 2, h2 is multiplied by ri and entered in row (i+1) for each i and so on. The contents of each row are added to give runoff qj, which is tabulated on Figure 11 (a). This also shows the plotted hydrograph, the net storm rainfall and the 0.4 hr. 10 mm unit hydrograph†

5.4.1 Base flow††

Adding an estimate of the base flow - or that portion of flow not included in runoff as described, must increase the runoff hydrograph computed. The Flood Studies recommend that this flow be estimated by the equation:

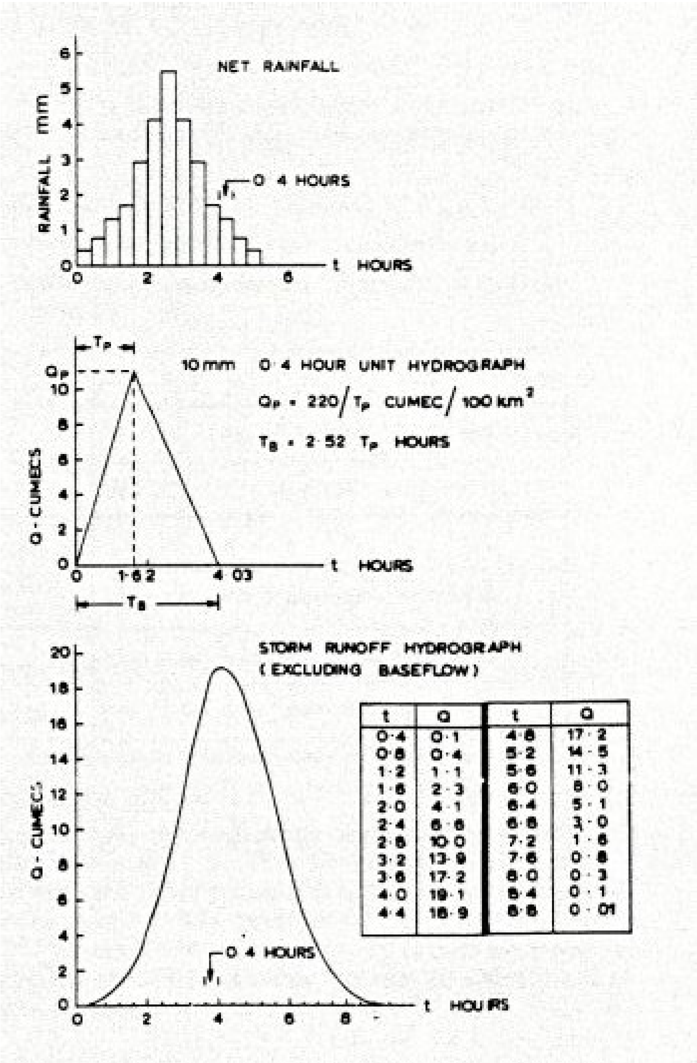
B = 0.00033 (CWI - 125) + 0.00074 R_{smd} + 0.003 cumec/km2............................(29)

In the example

B = 0.00033 (127 - 125) + 0.00074 (74.7) + 0.003 cumec/km2 = 0.47 cumec.

† Volume 1, p. 466 for direct calculation of peak if complete hydrograph not required.

†† Volume 1, chapter 6.5.11



t	Q	t	Q
0·4	0·1	4·8	17·2
0·8	0·4	5·2	14·5
1·2	1·1	5·6	11·3
1·6	2·3	6·0	8·0
2·0	4·1	6·4	5·1
2·4	6·6	6·8	3·0
2·8	10·0	7·2	1·6
3·2	13·9	7·6	0·8
3·6	17·2	8·0	0·3
4·0	19·1	8·4	0·1
4·4	18·9	8·8	0·01

Q_{25} ESTIMATION BY UNIT HYDROGRAPH METHOD

FIG 11a

The peak flow is therefore 19.1 1 + 0.47 = 19.58 cumecs. This then is the estimate of Q_{25} by the UH method. It will be noted that it differs markedly from the results of the statistical methods, based on the recorded data, in Section 4. This difference can be taken as an index of the possible error in flood estimates in the absence of flow data; had the hydrograph been computed using a unit hydrograph derived from the recorded data at the site much closer agreement could be expected from the results given by the two methods.

The Estimated Maximum Flood*

6.1 Introduction

The opinion is sometimes advanced that there is no upper limit to flood magnitude - only that probability of exceedance becomes smaller and smaller as the flood magnitude is increased, and similarly for rainfall magnitude. However, the meteorological studies (Volume II) showed that estimated envelopes of maximum rain on a rainfall-return period plot were in sufficient agreement to justify R_{max} values estimated by physical maximisation. With such estimates available (See Map 3} it is more readily possible to estimate the maximum flood. This is important in the design of reservoirs and spillways or in other circumstances where design failure would result in serious damage and loss of life.

(i)The recommended method of estimating the maximum flood is based on the unit hydrograph technique described in section 5, but with the following modifications:

T_P is reduced by a third because catchments tend to respond differently to extremely large storms that give peakier floods than moderate storms. Over all catchments studied the average ratio of observed minimum T_P to mean T_P was found to be two thirds.

(ii)The design storm duration, D, is also reduced by a third, by virtue of (i) and equation (27).

(iii)The design storm profile is taken as a symmetrical one of nested maxima, i.e. R_{max} is assumed to occur in every duration centered on the peak of the storm profile. The central interval of length τ contains τ Rmax, the central 3 intervals contain among them 3 τ Rmax and so on. The storm also includes a snowmelt allowance of 42mm/day = 1.75 mm/hr, up to a total equivalent to the expected maximum depth of snow[†].

(iv)A larger catchment wetness index is used which increases the percentage runoff and the base flow.

The estimation process is described in the following example, again using the Owengariff River at Tore Weir.

6.2 The unit hydrograph

Previously T_P was found to be 1.88 hours. In this case T_P = 2/3 (1.88) = 1.25 hours and a suitable data interval is τ = 1.25/5 = 0.25 hour. The one-hour UH is converted, approximately, to the 0.25 hour UH by altering T_P.

$$\text{New } T_P = T^1_P = \text{Old } T_P + \left(\frac{\tau - 1}{2}\right) = 0.875 \text{ hours}$$

Therefore the peak and base of the 0.25 hour UH are

Q^1_P = 220/0.875 = 251.4 cumec/100 km^2 = 20.11 cumec.

T_B = 2.52 x 0.875 = 2.2 hour.

The ordinates are calculated (Table 14) as before with s1= 20.11/0.875 = 22.98 and s2 = 20.11/(2.2 - 0.875) = 15.18. The first three ordinates are calculated using s1 while s2 is used for the remainder.

i =	1	2	3	4	5	6	7	8
t = it	0.25	0.5	0.75	1	1.25	1.5	1.75	2
hi =	5.75	11.49	17.24	18.22	14.44	10.66	6.88	3.1

Table 14. UH ordinates for maximum flood.

6.3 The design storm

This consists of both rainfall and a snowmelt contribution)
The duration D is

$D = (1 + R/1000)\ T^1_P = (1 + 2.335)\ 0.875 = 2.92$ hrs.

Since the profile is nested outwards from a central single value, it is convenient to have an odd number of data intervals. Therefore take 13 intervals, $\tau = 0.25$ and $D = 3.25$ hr. The maximum rainfalls for this locality (Map 4) are 2 hour $R_{max} = 135$ mm and 24 hour $R_{max} = 350$ mm. Using Table 3 the R_{max} values for durations between T = 0.25 hour and D = 3.25 hour are calculated and plotted against log of duration. A straight line smooths these and R_{max} values are read off for durations

τ =0.25hr, 3 τ = 0.75hr, 5 τ =1.25hr...

D = 13 τ = 3.25 hrs and entered in row 2 of Table 15. The quanities are necessary because of the requirement 6.1 (iii) above. They are then multiplied by the areal reduction factor, 0.96. The central 0.25-hour interval contains 53.8 mm gross and the central 0.75 hours contain 96 mm gross. Thus, the interval on either side of the peak contain (96-53.8) 12 = 21.1 mm gross. The half profile up to the central 7th interval is tabulated in row 5 and a snowmelt contribution of 0.44 mm (= 1.75 mm/hr x 0.25 hr) is added in row 6.

6.4 Percentage runoff

The catchment wetness index, CWI, is assumed to be 125 at a time 2D = 6.5 hr. before the beginning of the design storm. Over this 6.5 hours a total precipitation (rain and snow).

$P = 1/2\ (5D\ R_{max} - D\ R_{max}) + 2D \times 1.75$

$= 1/2(304 - 152) + 2 \times 3.25 \times 1.75 = 87.4$ mm.

is assumed. 5 D Rmax is obtained from Table 3 and is given in Table 15. CWI is now increased to

$CWI = 125 + P \times 0.^{52D/24} = 125 + 87.4 \times 0.5^{6.5/24} = 197.4$ mm.

Where the 0.5 is the daily decay factor in calculation of antecedent precipitation index. The percentage runoff is:

$P_r = 95.5G + 12U + 0.22\ (CWI - 125) + 0.1\ (D\ R_{max} - 10)$

$= 95.5\ (0.45) + 0 + 0.22(72.4) + 0.1\ (152 - 10) - 73\%$.

This is used to give the storm profile shown in the last row of Table 15.

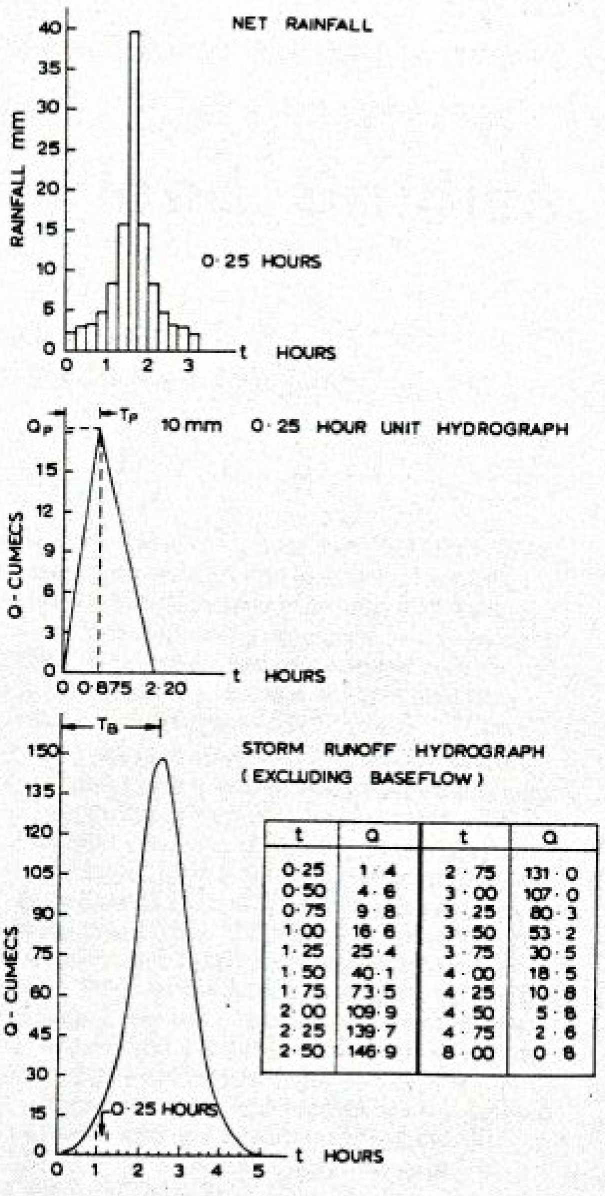
Duration Hours	0.25	0.75	1.25	1.75	2.25	2.75	3.25	16.25
R_{max}	56	100	122	135	144	152	158	317
$R_{max\ x\ 0.96}$	53.8	96	117.1	129.6	138.2	145.9	151.7	304.3
Profile Interval	1	2	3	4	5	6	7	8
Gross Rain, mm	2.9	3.85	4.3	6.25	10.55	21.1	53.8	
Gross Rain + snow, mm	3.34	4.29	4.74	6.69	10.99	21.54	52.24	
Net Rain + snow, mm	2.44	3.13	3.46	4.88	8.02	15.72	39.6	

Table 15. Calculation of design storm for maximum flood estimation.

6.5 <u>The storm runoff</u>

The design storm, unit hydrograph and resulting runoff are shown in Figure 11 (b). The convolution of net rainfall (cm) with the unit hydrograph is carried out as described in section 5.4. The peak runoff including base flow is 147.61 cumecs, which is the estimated maximum flood for the catchment. This discharge is about 25 times the mean annual flood and is 18.5 cumec/km^2.

*Volume 1, chapter 6.8
†Volume 2, chapter 7.4



t	Q	t	Q
0·25	1·4	2·75	131·0
0·50	4·6	3·00	107·0
0·75	9·8	3·25	80·3
1·00	16·6	3·50	53·2
1·25	25·4	3·75	30·5
1·50	40·1	4·00	18·5
1·75	73·5	4·25	10·8
2·00	109·9	4·50	5·8
2·25	139·7	4·75	2·6
2·50	146·9	8·00	0·8

ESTIMATION OF MAXIMUM FLOOD BY UNIT HYDROGRAPH METHOD

FIG 11b

Conclusion

The results contained in the Flood Studies Report mean that rainfall and flood estimation practice in this country have been put on a new footing. The discussion and examples presented show how to estimate rainfall and flood quantities of any return period for locations in Ireland, with or without recorded data at the site. An example of maximum possible flood estimation is also presented.

References

1)Flood Studies Report (1975), Natural Environment Research Council, 5 Volumes, 1198 pages and twelve maps (available from the Director, Institute of Hydrology, Maclean Building, Crowmarsh Gilford, Wallingford, Oxfordshire).

2)Watkins, L.H. (1962). 'The Design of Urban Sewer Systems" — Road Research Technical Paper No. 55. London: Her Majesty's Stationery Office.

3)Lloyd Davies, D.E. (1906) "The Elimination of Storm Water from Sewerage Systems". Proc. I.C.E. Vol. 164.

4)Institution of Civil Engineers (1933), "Interim Report of the Committee on Floods in relation to Reservoir Practice".

5)Institution of Civil Engineers, (1960), Subcommittee on Rainfall and Runoff. "Floods in the British Isles". Proc. I.C.E., Vol. 15.

6)Lynn, M.A. (1971), "Flood estimation for Ungauged Catchments". Transactions of Institution of Engineers of Ireland Vol. 96.

7)Institution of Civil Engineers (1966), "River Flood Hydrology". Proc. of Symposium held at I.C.E. in March 1965, London.

8)Institution of Civil Engineers (1967), "Flood Studies for the United Kingdom". Report of the Committee on Floods in the United Kingdom..

9)Langbein W.B. (1949) "Annual Floods and the Partial Duration Series". Trans, Amer Geophys. Union Vol. 30.

10)Gumbel E.J. (1941) "The Return Period of Flood Flows", Annals of Math. Statistics, Vol. 12.

11)Dalrymple T. (1960) "Flood Frequency Analysis". U.S. Geological Survey Water Supply Paper 1543 A, U.S. Govt. Printing Office, Washington.

Acknowledgements

The authors wish to record their thanks to the staff of the Office of Public Works who helped in the production of the paper and in particular to Messrs. J. Howard B.E., C.Eng. J. Martin B.E., M.Eng. Sc. and T. Bree B.E., M.Sc., for their special assistance. They also wish to thank the Chief Engineer, Mr. S. Shiel 8.E., B.Sc., C.Eng, F.IEI, and the Commissioners of Public Works for permission to present the paper.