Estimating Ireland's output gap; an analysis using selected statistical filters

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An Roinn Airgeadais
Department of Finance

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ACKNOWLEDGEMENTS

We would like to thank Adele Bergin, Eddie Casey, Feargal O'Brolcháin, Brendan O'Connor and Graeme Walsh for their comments. We thank Laura Weymes for her advice and provision of earlier material involving the design of a multivariate Kalman filter that accounts for financial cycle impacts. We also benefited from discussions with the IFAC Council members and Central Bank of Ireland staff members. The analysis and views set out in this paper are those of the authors only and do not necessarily reflect the views of the Department of Finance or the Minister for Finance.

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Executive Summary

This paper presents the statistical filter models used by the Department to estimate the cyclical position (i.e. output gap) of the Irish economy, which were first published in *Stability Programme Update 2018* and then subsequently as part of *Budget 2019*. Estimates of the output gap serve as an important input into the fiscal policymaking process and the macroeconomic management of the economy. They enable the Department to better evaluate the appropriate fiscal stance and the sustainability of public finances over the medium term.

Previously, the Department's main statistical approach to estimating the economy's cyclical position has been the European Commission's harmonised approach. However, as has been well documented on many occasions by the Department and the Irish Fiscal Advisory Council this approach suffers from a number of limitations which have provided counter intuitive estimates of Ireland's cyclical position. The Department contend that such estimates should not be solely relied upon for assessing the economy's stance in the context of macroeconomic management and fiscal planning.

As discussed in the literature review (Murphy et al., 2018), which accompanies this paper, there is no standard approach to estimating potential output or the output gap. From a statistical modelling perspective, many valid alternative technical assumptions can be made on how best to identify the level of potential output. However, alternative modelling procedures can produce quite different estimates. In that respect, it is prudent to develop a range of approaches.

Accordingly, the Department applies a number of modelling techniques including the univariate and multivariate Kalman filter, the Hodrick Prescott (HP) filter and the extended HP filter in the spirit of Borio et al. (2013, 2014). We include a range of explanatory variables that provide additional insight into the economic cycle in Ireland such as the unemployment rate, net migration, credit growth and house prices. Given recent distortions in real GDP, domestic GVA is also used in the analysis as an alternative measure of economic activity in Ireland. Estimates from these models were then assessed by examining a number of factors including the plausibility of output gap paths, real time estimate volatility and performance of the estimates in explaining inflation.

In terms of main findings, the output gap estimates (i.e. based on midrange of the GDP based estimates) suggest a modest negative output gap in 2018 consistent with limited inflationary pressures in the economy and remaining slack in the labour market. The estimated output gap is forecast to turn slightly positive in 2019 and increases thereafter, pointing to signs of overheating over the medium term.

To the extent that real time estimates are subject to revision it is important that they are not entirely relied upon for the purposes of fiscal policy making. In this

regard, these statistical estimates represent an important additional input to the Department's overall approach to assessing the economy's stance.

Finally, the Department intends to build upon this work with consideration being given to the application of semi structural and structural models. Such models can facilitate a more in-depth analysis of future developments in potential output and the output gap.

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Section 1 – Introduction

This paper presents the statistical filter models used by the Department to estimate the cyclical position of the Irish economy (i.e. the output gap). Estimates of the output gap serve as an important input into the fiscal policymaking process and the macroeconomic management of the economy. They enable the Department to better evaluate the appropriate fiscal stance and the sustainability of public finances over the medium term.

However, accurately measuring the economy's cyclical position is quite a formidable task. Notwithstanding the conceptual issue of defining the potential output of a small open economy such as Ireland, which is characterised *inter alia* by an elastic labour supply and cross border mobility of capital, there remains the empirical challenge of selecting an appropriate statistical model to estimate a variable which is not directly observable. Furthermore, distortions to the Irish national accounts related to activities of some multinational national firms, along with large data revisions provide an additional layer of complexity.

Heretofore, the Department's main statistical approach to estimating the economy's cyclical position, has been the European Commission's harmonised production function methodology (harmonised approach).[1] These estimates are required for the purposes of assessing compliance with the preventive arm of the Stability and Growth Pact (SGP). However, this methodology suffers from a number of well documented shortcomings which have occasionally provided counter intuitive estimates of Ireland's cyclical position. The mobility of the factors of production can lead to pro-cyclical estimates of potential output (i.e. potential output growth closely tracks actual output growth) while there is also doubts over the suitability of the approach used estimate the natural rate of unemployment (NAWRU), the measurement of capital stock and total factor productivity (TFP). In more recent years, the methodology has struggled to deal with distortions in Ireland's GDP.

Consequently, the Department and more recently the Irish Fiscal Advisory Council contend that such estimates should not be solely relied upon when assessing the economy's stance in the context of macroeconomic management and fiscal planning. It follows that there is strong merit in developing alternative approaches which can produce more intuitive output gap estimates. Accordingly, this paper reflects the Department's commitment to the ongoing development of more plausible (i.e. output gap estimates that better reflect the Department's view of the cyclical position of the economy) supply-side estimates.

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[1] Since 2015, the public finances in Ireland are subject to the preventive arm of the Stability and Growth Pact (SGP) which has given greater prominence to potential output and the output gap in Irish fiscal policy. A key requirement of SGP is that Member States meet or progress towards their Medium-Term Budgetary Objective of a balanced budget in structural terms, thus requiring the estimation of the cyclically adjusted budget balance.

To this end, the Department has undertaken a review of the related literature for the purposes of identifying appropriate statistical models (Murphy, Nacheva and Daly, 2018). A key point from the review is that there is no standard approach to estimating potential output and in turn the output gap. Moreover, and of particular relevance to policymakers, is that alternative modelling approaches can produce quite different estimates. This uncertainty can be explained by a number of factors including for example, data and forecast revisions, model and parameter uncertainty.

In view of these findings, the Department has adopted a prudent approach and set about applying a number of statistical methods to gauge the economy's cyclical position. This analysis is organised under a three pillar framework. Under the first pillar, the Department will continue to apply the harmonised European Commission-approved method for the purposes of calculating certain cyclically adjusted fiscal measures. Under the second pillar, an alternative production function will be developed with the aim of addressing the main shortcomings of the harmonised approach as they relate to Ireland.[2] The third pillar, which is the focus of this paper involves the application of other statistical methodologies.

The first set of models included under Pillar 3 that are presented in this paper consist of a number of univariate and multivariate filtering methods. More specifically, the set of approaches considered here include the univariate and multivariate Kalman filter, the Hodrick Prescott (HP) filter and the extended HP filter model approach developed by Borio et al. (2013). These statistical filter models were selected for a number of reasons;

- they are widely used, for instance, by researchers and central banks to calculate trend components (e.g. potential output),
- the models are relatively simple to implement with limited constraints imposed on the data,
- they provide a genuine alternative to the harmonised approach,
- they will be complemented by additional models over time; the Department intends to develop more in-depth models which better account for the key drivers of potential output.

One issue that is given much consideration is the selection of output measures that appropriately reflect economic activity in Ireland. In recent years, questions have been raised about the economic relevance and volatility of GDP due to, for instance, the growing prevalence of contract manufacturing and relocations of intellectual property assets. Taking cognisance of this issue, we follow Casey (2018) and IMF (2015a; 2015b) and consider a measure of 'domestic' economic output (i.e. domestic GVA) in addition to GDP. This series excludes the output from multinational dominated sectors, which the authors contend provides a more appropriate estimate of the Irish economic cycle. Moreover, Casey (2018) finds that developments in domestic GVA are more relevant for explaining changes in Irish tax receipts. However, multinational dominated sectors are defined as those where more

[2] It is envisaged that this work will form the basis of future submissions and engagement with the EU Economic Policy Committee's Output Gap Working Group (OGWG) on the inclusion of economically justified Irish specific amendments to the harmonised approach. This work will be published in a separate paper at a later stage.

than 85 per cent of gross output is produced by foreign owned firms. By excluding these sectors in construction of the domestic GVA variable means that some domestic activity is excluded. Similarly, for those sectors defined as being domestic dominated sectors, some element of MNE gross value added is included. For this reason we consider both output measures.

In assessing estimates from these models a number of factors including the plausibility of output gap paths, real time estimate volatility and performance of the estimates in explaining inflation were considered. Our preferred models show that financial and labour market variables contain important information about Ireland's business cycle fluctuations. These variables appear to better explain changes in the magnitude of the output gap over time compared with their respective univariate counterpart. The incorporation of these variables demonstrates the importance of looking beyond aggregate capacity constraints and accounting for the potential misallocation of resources and the build-up of sectoral imbalances.

Our findings suggests that the output gap (i.e. based on midrange of GDP based estimates) will turn positive in 2019 and continue to widen by 2023. Such suggestions of overheating are consistent with forecasted developments in the model variables including unemployment rate, migration and construction employment.

The paper is structured as follows; section 2 begins with a discussion of the data that is used in the analysis with particular attention being paid to the various transformations and forecasting assumptions applied to the data. In section 3 the alternative filtering models and output gap estimates are presented. Section 4 evaluates the alternative estimates and provides an assessment of the cyclical position. Section 5 concludes the paper and outlines avenues for future work.

Section 2 – Data

We begin by providing a detailed description of the data used and all relevant transformations before describing in a subsequent section the processes and assumptions underlying each model along with the selection procedure used to identify the preferred models.

2.1: dependent variables

For the reasons discussed above, two measures of output have been selected; quarterly real GDP and annual real domestic GVA. The real GDP data are taken from the Quarterly National Accounts for the period 1995-2017.

The domestic GVA series is taken from the annual National Income and Expenditure accounts (NIE) published by the CSO in 2018. The data are available on an annual basis for the period 1995-2017. To extend the series back to 1970, we follow the approach in Casey (2018) and link domestic GVA with GNP through the following relationship;

$$\Delta Domestic\_GVA_t = \alpha + \beta \Delta GNP_t + \varepsilon_t \qquad (1)$$

The model parameters are estimated using ordinary least squares based on a sample for the period 1995-2017 (with variables in log-levels).

2.2: explanatory variables

In the context of the multivariate models, we consider the inclusion of a range of variables in the models which the literature has identified as being relevant for explaining the economic cycle.[3]

2.3: financial and housing market variables

Following Borio et al. (2014), we look to account for the impact of the financial cycle in measures of potential output since it is possible for imbalances in this sector to accumulate without an inflationary response. We consider real private sector credit growth and real house price growth. Together, these capture the interaction between financing constraints, collateral values and wealth effects (Kiyotaki and Moore, 1997). Consideration was also given to the short-term real interest rate, which subject to the response function of the central bank, should move counter-cyclically (Borio et al, 2013). However, as discussed below, the variable was not included in the analysis as it did not comply with the necessary model assumption of mean stationarity.

We also consider the share of employment in the construction sector (Orlandi, 2012) and investment in building and construction to capture trends in the housing sector.

2.4: labour market variables

As a small open economy with a relatively elastic labour supply, it would appear to be particularly important to account for the role of migration over the economic cycle. Migration is often regarded as a ‘safety valve’ for the Irish economy; when the economy is growing strongly net inward migration can alleviate production constraints while outward migration can assuage economic slack during downturns. A recent IMF paper (IMF, 2018) identifies a high positive correlation between changes in Ireland’s labour force and employment suggesting that migration is driven by the availability of employment. To account for the role of migration over the business cycle, net migration data is used in the analysis. Consideration is also given to the unemployment rate and the employment ratio (relative to the working age population) to account for cyclical developments in the labour market.

2.5: other explanatory variables

The current account is another relevant factor when accounting for the cycle (Darvas and Simon, 2015; Casey, 2018). Bénétrix and Lane (2015) highlight that it can often be used as a way to identify the absorption cycle. However, the headline current account balance is particularly distorted in recent times by activities of some multinational firms facilitated by greater mobility of international assets, aircraft leasing and capital. We consider the modified current account which seeks to provide a more meaningful measure. It is defined as the current account less depreciation on R&D service imports and trade in intellectual property (IP), redomiciled incomes and aircraft leasing depreciation adding back net aircraft related to leasing, R&D related IP imports and exports and R&D service imports. A number of measures of inflation are also considered including core CPI as indicators of the cycle, Rünstler (2002).

[3] A number of variables are only available at an annual frequency, in order to facilitate their inclusion in the models where quarterly real GDP is the dependant variable, a quadratic interpolation was applied to produce a quarterly series.

Table 1: Variables considered

Variable	Description	Source
Economic Output		
Real Gross Domestic Product	Quarterly , 2016 prices	CSO
Domestic Gross Value Added (GVA)	Annual, 2016 prices	CSO
Financial Variables		
Real Private Sector Credit Growth	First difference	CBI
Real House Price Growth	First difference	BIS
Real Investment in Building and Construction	First difference; share of real GDP^	CSO
Real Interest Rates		OECD
Core CPI	CPI excluding energy and unprocessed food	CSO
GDP Deflator		CSO
Adjusted Current Account	% GNI*	CSO
Labour Market Variables		
Unemployment Rate	% Labour Force	CSO
Net migration	% Labour Force	CSO
Employment to Working Age Population	Ratio	CSO
Construction Sector Employment	Share of total employment	CSO

Note: Data for two variables (i.e. adjusted current account and migration data) are only available at an annual frequency so in order to facilitate their inclusion in the models where quarterly real GDP is the dependent variable, a quadratic interpolation was applied to produce a quarterly series. In line with Borio et al. (2013), private sector credit excludes credit extended for financial intermediation, since the financial sector is strongly driven by international as opposed to domestic factors.
^To correct for the level shift in Q1 2015, we assume a y-o-y growth rate in Q1 is equal to the average y-o-y growth rate of Q2-Q4 2015.
Source: CSO, CBI, BIS, OECD.

2.6: stability of the means

An important modelling assumption is the mean-stationary of the explanatory variables. Similar to Borio et al. (2014), we check for mean stationarity by using an extended window average.[4] We first calculate the mean based on data ranging from 1990Q1 to 1995Q1, and then successively increase this sample by adding additional observations (one at a time), whilst keeping the start date fixed. As illustrated in Figure 1, with the exception of real interest rates, which have exhibited a downward trend for most of the period since the 1990s, the means of all variables appear to be stationary. To transform the data, we take logs and smooth the data by taking a four-quarter moving average before de-meaning all explanatory variables. Variables which are expressed as shares are not logged. These include the unemployment rate, migration, employment in construction and the employment to working age population.

[4] Borio et al (2014) discuss in detail the relevance of mean stationary and its implications for extended HP model estimates.

Figure 1: Mean stationary of explanatory variables

Note: The mean is calculated based on an expanding sample. The mean in 1995Q1 is based on the sample from 1990Q1 to 1995Q1. It is recalculated in each subsequent period as an additional observation is added to the sample.
Source: Department of Finance calculations

2.7: forecasting of explanatory variables

One of the key purposes of developing the alternative methodologies is to produce forecasts of potential output and the output gap. To this end, for each variable - when applicable - we use the relevant forecast for the period 2018 to 2023 as prepared for Budget 2019. However, it is necessary to forecast the variables by an additional three years to 2026 in order to reduce the end-point bias of the model estimates in 2023. The variables are extended by a further three years using ARIMA forecasting. Therefore, the variables are forecast approximately nine years ahead from the last outturn. A summary of the forecasting approach for each variable is included in Table 2.

Table 2: Forecasting of variables

Variable	Description of forecast procedure
GDP	We take Budget 2019 annual forecasts to 2023. To construct the forecasted quarterly series, a constant quarterly growth rate is assumed which is consistent with the respective annual forecast.
Domestic GVA	GNP forecasts to 2023 are used to project domestic GVA based on the following relationship $\Delta Domestic\_GVA_t = \alpha + \beta \Delta GNP_t + \varepsilon_t$. Parameters estimated using OLS on outturn data. Foreign GVA is calculated as the difference between total GVA and domestic GVA where GDP growth forecast are used to forecast total GVA.
House Prices	These forecasts are judgement based. Annual house price growth is forecast to slow to 3.4% by 2023.
Private Sector Credit Growth	Growth is assumed to revert to long-run rate by 2023.
Unemployment Rate	We take Budget 2019 annual forecasts to 2023. For extended HP filter, a constant quarterly growth rate is assumed which is consistent with the respective annual Budget 2019 forecasts.
Net Migration	We take Budget 2019 annual forecasts to 2023. For 2024 -2026, we take the CSO's long-term projections of the change in migration as a share of the labour force (scenario M1, which assumes net inflow of 30,000 per annum).
Construction employment	Budget 2019 annual forecasts to 2023.
Core CPI	Budget 2019 annual forecasts to 2023.

Note: Eviews automatic ARIMA forecasting function is used to forecast model variables for the period 2024 to 2026.
Source: Department of Finance

2.8: measuring the output gap

Before proceeding it is necessary to discuss how the output gap is calculated. For GDP based model estimates, the standard definition of the output gap is as follows:

$$Output\ Gap_{gdp} = \left(\frac{\text{GDP} - Trend\ GDP}{Trend\ GDP}\right) * 100 \qquad (2)$$

We now turn to examine the GVA based output gap estimates in the usual formulation but here we disaggregate the total GVA output gap into a domestic and foreign components:

$$Output\ Gap_{gva} = \left(\frac{\text{Domestic } GVA - Filtered\ (\text{Domestic } GVA)}{Filtered\ (\text{Domestic } GVA)} + \frac{\text{Foreign } GVA - Filtered\ (\text{Foreign } GVA)}{Filtered\ (Foreign\ GVA)}\right) * 100 \qquad (3)$$

Following the literature, a standard assumption to make is that these Foreign or MNE sectors operate at full potential or fully utilise the resources available to them. In other words, the output gap for the foreign MNE-dominated sectors (i.e. $Foreign\ GVA - Filtered\ (Foreign\ GVA)$) is assumed to be equal to zero. This formulation allows one to measure the performance of domestic economic activity relative to the economic potential of the Irish economy as a whole. The domestic GVA output gap is now calculated as follows:

$$Output\ Gap_{gva} = \left(\frac{\text{Domestic } GVA - Filtered\ (\text{Domestic } GVA)}{Filtered\ (\text{Domestic } GVA) + Foreign\ GVA}\right) * 100 \qquad (4)$$

Section 3 – Application of statistical filter approaches

Two statistical filters are used in the analysis, namely the extended Hodrick-Prescott (1997) filter model and the Kalman filter. A brief exposition of each methodology is provided followed by a discussion of the estimation strategy applied and the model estimates.

3.1: univariate HP filter

The HP filter assumes that a given time series is the sum of a trend component and a cyclical component.

$$y_t = {y^*}_t + e_{0,t} \tag{5}$$

$$\Delta {y^*}_{t+1} = \Delta {y^*}_t + e_{1,t} \tag{6}$$

The decomposition process is framed by a loss function, whereby the filter chooses estimates of potential output which minimise the sum of the variance of its rate of growth ($e_{1,t}$) and the cyclical component (e_0).

$$Min \sum_{t=1}^{T} \left[\frac{1}{{\sigma^2}_0} (e_{0,t})^2 + \frac{1}{{\sigma_0}^2} (e_{1,t})^2 \right] \tag{7}$$

The relative weight placed on the cycle and rate of potential growth is pre-determined and represented by the parameter λ:

$$\lambda = \frac{e_0^2}{e_1^2} \tag{8}$$

The detrending parameter λ plays a crucial role in the estimation process as it determines how smooth the trend will be (i.e. how much volatility of the cycle will be removed). The higher the value of λ the more volatility that is removed from the trend component. In the limit where λ is equal to infinity, the extracted trend approaches a linear trend. As discussed in the background paper (Murphy et al., 2018), this parameter has also given rise to strong criticism of the HP filter because the appropriate value must be judged by the researcher and the results can be sensitive to the value selected.

For the univariate HP model, λ is set equal to 1600. Whilst debate around the appropriate value of λ for annual data is ongoing, there appears to be greater consensus around the value of 1600 for quarterly data (Ravn and Uhlig, 2001; Cooley and Ohanian, 1991). Setting λ to 1600 assumes a business cycle duration of approximately 8 years.

3.2: extended HP filter

The univariate HP filter can be modified to incorporate exogenous variables, resulting in an extended (i.e. multivariate) HP filter. The clear advantage of this filter over its univariate counterpart is that the inclusion of exogenous variables can help to inform the decomposition process, and as a result, improve the performance of the model.

To extend the HP filter, the cycle component of output is re-expressed, whilst the dynamics of the trend remain unchanged. Therefore, within the state space system of equations, adjustments are made only to the signal equation (eq. 9). This entails the inclusion of a set of exogenous variables denoted by the matrix X which are intended to capture information on the behaviour of the cycle component. Second, the output gap is assumed to follow an autoregressive process (of an order of one), as opposed to a random one.

As noted above, there is much discussion on the extent to which developments in GDP accurately captures domestic economic activity in recent years. In this context, we account for the level shift in GDP in Q1 2015 due to firms' assets relocation and the increase in GDP in Q4 2016 due to the one time increase in intangible investment.[5] Within the state space framework, this is accounted for with the inclusion of pulse dummies in the state equation (eq. 10).

$$y_t = {y^*}_t + \beta\left(y_{t-1} - {y^*}_{t-1}\right) + \gamma' X_t + e_{2,t} \tag{9}$$

$$\Delta {y^*}_{t+1} = \Delta {y^*}_t + e_{3,t} \tag{10}$$

The corresponding loss function is as follows:

$$Min \sum_{t=1}^{T} \left[\frac{1}{{\sigma^2}_2}(e_{2t})^2 + \frac{1}{{\sigma_3}^2}(e_{3,t})^2\right] \tag{11}$$

In this alternative specification, the volatility of the cycle is no longer exclusively determined by the smoothing parameter λ; it is also a function of the auto-regressive parameter β and the vector of coefficients γ. In order to preserve assumptions around the frequency of the business cycle, the value of λ is solved for in order to ensure that the variance of the cycle relative to the trend growth rate is equivalent to that of the univariate filter:

$$var\left(\frac{e_0}{e_1}\right) = var\left(\frac{e_2}{e_3}\right) \tag{12}$$

In order to ensure that the output gap is stationary and has a mean of zero, we restrict β to below 0.95.[6] In addition, we must de-mean the explanatory variables as the inclusion of a trending variable would cause the variance of the output gap to increase over time. More specifically, if a variable with a non-stationary mean was included then potential growth would need to increase in order for equation 12 to hold. However, this would also increase the loss function, implying that the minimising solution is to set the coefficient of the trending variable to zero.

As with the univariate filter the results are conditional on the value of the smoothing parameter λ. However, with the inclusion of financial variables, a further complicating issue arises which relates to differences in the duration of financial cycles and business cycles. With the HP framework the standard assumptions around the duration of the business cycle imposes a shorter duration on the financial cycle that is inconsistent with both the theoretical and empirical literature. Borio et al. (2013) acknowledge this limitation in their paper. The financial cycle (often 16 to 20 years) tends to be longer than the typical business cycle. As such, when using this procedure, longer term trends present in the financial data may be

[5] To correct for the level shift in Q1 2015, we assume a y-o-y growth rate in Q1 is equal to the average y-o-y growth rate of Q2-Q4 2015.

[6] The results are not sensitive to the value of the upper bound.

misinterpreted as cyclical. This would entail a higher potential output and a lower output gap than it would otherwise be.

Berger et al. (2015) attempt to address this issue by pre-filtering the variables (using a HP filter) before incorporating them into the model.[7] They account for the long frequency of the financial cycle, by setting λ at a value of 63,500 (which excludes movements at frequencies of 20 years or longer). This is intended to prevent any existing longer-term financial trends being misinterpreted as cyclical in nature. We have attempted to replicate this approach but find that pre-filtering eliminates the short-term volatility of the explanatory variables and they become insignificant. To the extent that financial variables carry useful information we continue to include them in the model while remaining cognisant of potential output gap measurement issues.

In addition to financial variables there are many variables which may contain relevant information about the business cycle. As noted in Borio (2014), there may be gains from combining variables in the model, although care must be taken in regards to the application of statistical and economic criteria. Accordingly, a range of variables identified by the literature as being relevant for explaining the economic cycle have been considered. We pay particular attention to financial and labour market variables given their historical relevance in explaining Ireland's business cycle. The next section outlines the process undertaken to select the appropriate model specifications.

3.3: model selection

A series of steps were taken before selecting our preferred models. First, two models containing labour market variables and another three models with capital variables were specified. In the interest of parsimony, each model includes a single lag of the respective explanatory variables.[8] Various permutations of the explanatory variables lags were then estimated. The preferred models were selected having inspected the output gap estimates, the significance of the explanatory variables, AIC and log-likelihood statistics.[9]

The preferred labour market model includes the unemployment rate and net migration as a share of labour force as explanatory variables. The preferred financial model includes real private sector credit growth, employment in the construction sector and core CPI.[10]

3.4: extended HP estimation results

The regression results show that both the financial and labour market variables contain important information about Ireland's business cycle fluctuations (Table 3). In the case of the capital model (HP_K) all variables are significant and the signs are in line with expectations. The output gap is found to follow an autoregressive process, with a slow rate of decay. The results for the labour model, show that both explanatory variables are significant with the expected signs. The coefficient on the output gap is also significant and of a similar magnitude to the first model.

[7] As the authors note, the disadvantage of this approach is that the input data would be subject to an end-point bias.
[8] This approach is consistent with the one taken by Borio et al. (2014).
[9] A range of criteria to assess performance of real time estimates are discussed in detail in section 4.
[10] A description of the alternative models that are estimated is provided in the Appendix.

Table 3: Extended HP filter model estimates

	Capital model (HP_K)	Labour model (HP_L)
Output Gap (-1)	0.80***	0.82*
	(0.09)	(0.07)
Private Sector Credit Growth	0.30**	
	(0.12)	
Construction employment share	1.08*	
	(0.40)	
Core CPI	1.15**	
	(0.39)	
Unemployment Rate		-0.23*
		(0.14)
Net migration share		7.57***
		(1.08)
Observations	128	128
AIC	4.21	4.07
Log Likelihood	-236.35	-251.86

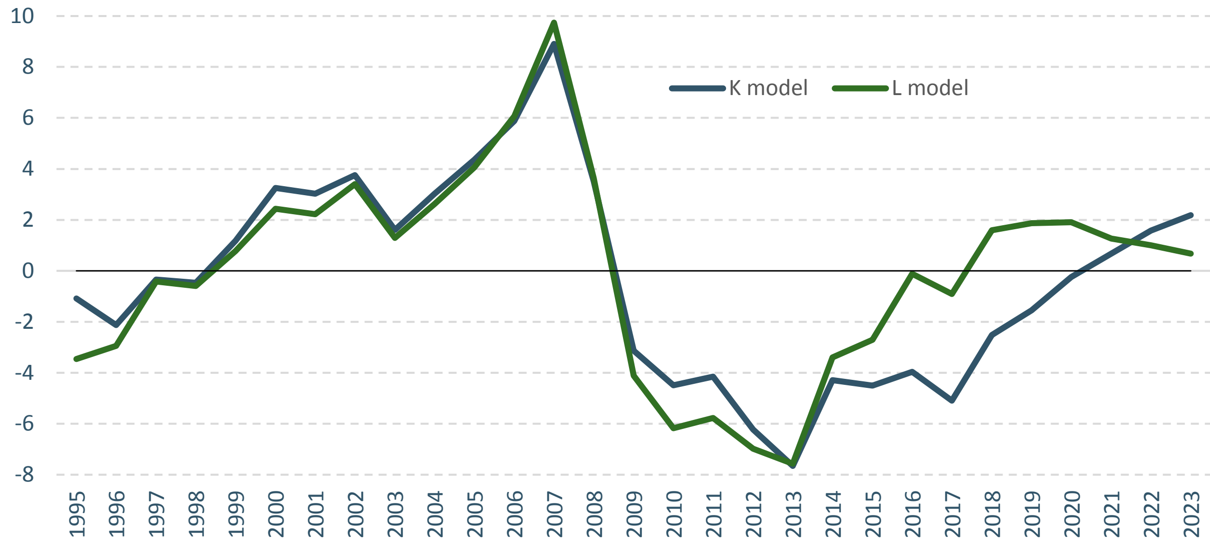
Note: Dependent variable is quarterly real GDP. Dummy variables included to account for Q12015 and Q42016 GDP distortions are not presented. Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1)
Source: Department of Finance calculations

The two models produce very similar estimates up to 2013 as illustrated in Figure 2. The output gap turned positive in the late 1990s and widened in almost every year thereafter until it peaked in 2007 at around 9 per cent. Whilst the models indicate somewhat varying output gap paths following the onset of the crisis, both models suggest a trough of approximately -8 per cent in 2013. This implies a peak-to-trough range of around 17 percentage points, which is in line with alternative estimates, such as those produced by the OECD. This finding is encouraging, considering the general uncertainty associated with estimates of the economic cycle and methodological differences across the models.

Interestingly the output gap paths begin to diverge after 2013. This divergence is largely consistent with the developments in the explanatory variables included in the respective models. In this regard, it is important to bear in mind that the output gap path over the forecast horizon is underpinned by the expected (forecasted) trends in the explanatory variables (plots of these variables are located in Figure C1 in Appendix C).

The capital model indicates a negative output gap of around -5 per cent in 2017. This reflects the trends in the explanatory variables such as the deleveraging in the private sector, the lack of inflation and subdued employment in construction up to this point. However, the output gap turns positive in 2021 before quickly widening to over 2 per cent by 2023. This is consistent with the forecasted increase in private sector credit growth, stronger inflation and a larger share of employment in construction.The labour model suggests the output gap turns positive in 2018 jumping to 1.7 per cent reflecting the forecasted increase in the migration share and lower unemployment rate. There appears to be some closure of the output gap from 2021 onwards which is driven in part by the forecasted drop in migration attributed to Brexit and the marginally lower net inflow thereafter.

Figure 2: Output gap estimates using extended HP filter, % of Potential Output



Source: Department of Finance calculations

3.5: Kalman filter

We now focus our attention on the application of the Kalman filter to provide estimates of the output gap using annual domestic gross value added data.[11] The Kalman filter is an algorithm which is used to calculate minimum mean square error forecasts in state space models. Put simply, the filter is a recursive process which sequentially updates its one step ahead mean and variance forecasts every time a new observation becomes available. The approach is increasingly used in the trend-cycle decomposition literature and is now discussed in more detail.

3.6: unobserved components decomposition

Using the state-space format, we define a signal equation which relates the observed output measure Y_t, and its unobservable components: trend output Y^* (expressed in log levels) and the output gap OG_t.

$$Y_t = Y_t^* + OG_t \tag{13}$$

The state equations describe how the components evolve over time. These components, for instance, can be modelled explicitly as deterministic functions or stochastic processes. By way of example, the following set of state equations model trend output (equation 14) as a random walk with time varying drift (u_t) with shocks e_t that are normally distributed. The drift term - as specified in equation 15 - in turn is assumed to follow a random walk with a disturbance term γ_t. The output gap is modelled in equation 15 as a stationary first order autoregressive process AR(2). [12]

[11] The Department in earlier work developed a multivariate Kalman filter to incorporate financial cycle impacts, Weymes (2016).

[12] For the purposes of clarity, the 'output gap' OG_t is specified in levels.

$$Y_t^* = u_t + \beta_1 Y_{t-1}^* + e_t \tag{14}$$

$$u_t = u_{t-1} + \gamma_t \tag{15}$$

$$OG_t = \beta_2 OG_{t-1} + \beta_3 OG_{t-2} + \beta_4 X_{t-i} + \epsilon_t \tag{16}$$

In the multivariate case, relevant explanatory variables X_{t-i} are included in the output gap state equation. To the extent that these variables better define the evolution of the output gap, the model indirectly accounts for their impact on potential output. The above model and a number of augmented variants are estimated using annual domestic GVA data for the period 1975 to 2023.[13]

The Kalman filter works in two stages. In the first stage, the Kalman filter produces estimates of the current state variables along with their variances. This is based on observed signal variables and assumptions imposed by the researcher regarding the mean and variance of the unobserved variables. In the second stage, once the initial projection of the state variable is observed, these estimates are updated using a weighted average, with more weight being given to estimates with higher certainty or lower variance. The model can be estimated using maximum-likelihood as we have historical data on the observable series Y_t, and we further assume that the state variables and the error term are normally distributed.

Within the state space model framework, the initial values for the state variables and disturbance variances tend to be unknown and must be determined either by the researcher or within the model (i.e. diffuse initialisation). For our purposes, the starting values for the output trend and output gap state equations (i.e. Y_0^* and OG_0 respectively) are the 1975 values of the HP filtered trend and cycle estimates derived from the actual economic output series (e.g. real domestic GVA).[14] The initial value of the variance of the disturbance term in the trend state equation (i.e. e_0) is determined within the model. The initial value of the disturbance terms variance specified in the output gap equation (i.e. ϵ_0), is calculated as the variance of the residuals taken from an OLS estimation of the equation based on the HP derived output gap series.[15] The error variance term in the drift equation is calculated in a similar fashion. The initial variance-covariance matrix is also based on the HP filtered output. The model parameters are estimated using maximum likelihood.

3.7: model selection

The unobserved components decomposition is quite flexible in the application of hypothesised structures for the trend and cycle components. A number of alternative specifications were estimated. The suitability of each model was assessed based on a number of criteria including the plausibility (i.e. path of the output gap) of the output gap estimates, examination of auxiliary residuals, the significance of the key parameters, the model fit based

[13] Forecasts are included for the period 2018 to 2023. The log-level of actual economic output is scaled by 100 before estimation in order to obtain convergence.
[14] HP filter was applied to series over the period 1970 to 2023.
[15] We assume a lambda of 6.25 when applying the HP filter.

on the AIC, and diagnostic tests for independence, homoscedasticity and normality where applicable.[16] The preferred model specifications are listed in Table 4.

Table 4: Preferred model specifications (based on domestic GVA)

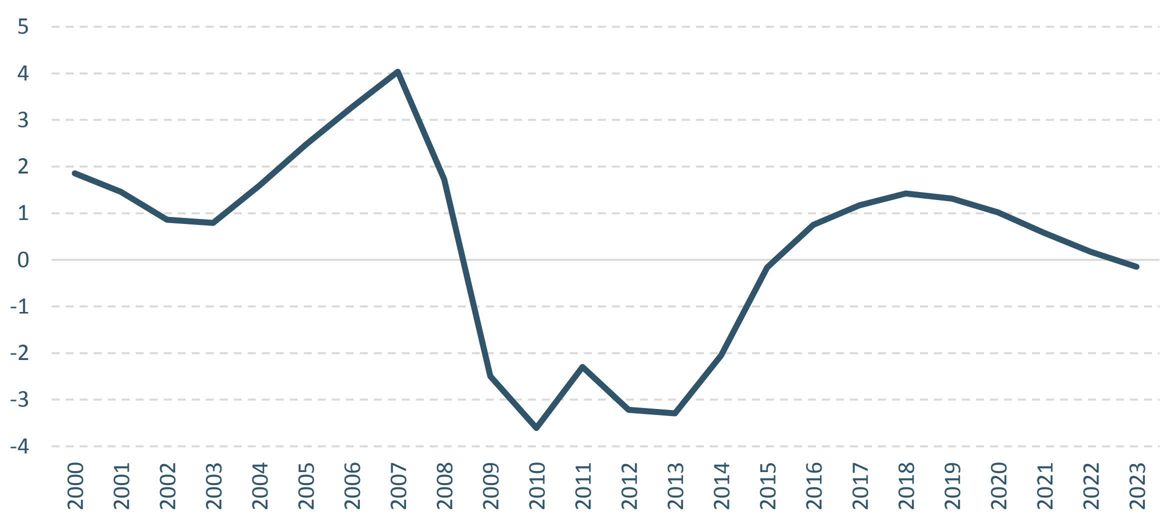
Model		Specification
1	KF	KF: Trend; random walk with drift cycle; AR(2)
2	KF_hp	Multivariate KF: Trend; random walk with drift, cycle; AR(2), House prices growth
3	KF_pscg	Multivariate KF: Trend; random walk with drift, cycle; AR(2), Private sector credit growth
4	KF_u	Multivariate KF: Trend; random walk with drift, cycle; AR(2), Unemployment rate
5	KF_c	Multivariate KF: Trend; random walk with drift, cycle; AR(2), Construction employment share

Source: Department of Finance

3.8: estimation results

The output gap estimates based on the univariate Kalman filter model suggest a largely plausible narrative of Ireland's business cycle over the period 2000-2017. It captures the lead up to the peak in 2007 and subsequent sharp decline consistent with the impact of the financial crisis. This is followed by a number of years of a relatively stable output gap before beginning to close from 2013 onwards. This initial closure of the output gap is broadly reflective of declining slack in the labour market at the time. However, the return of the output gap to positive territory in 2016 is subject to debate given the continued slack in the labour market (e.g. unemployment rate stood at 8.4% in 2016) and limited inflationary pressure. The output gap widens to 1.4 per cent in 2018 before it begins to close again turning marginally negative by 2023. As regards diagnostic tests and inference, output from the model are presented in Table 5. Inspection of the standardised prediction errors for independence, homoscedasticity and normality suggest the assumptions are largely satisfied.

Figure 3: Domestic GVA based output gap estimates using univariate Kalman filter



Source: Department of Finance calculations

[16] Commandeur and Koopman (2007) and Van de Bossche (2011) provide useful discussions on the application of these diagnostic tests.

Table 5: Selected parameters and diagnostics for univariate Kalman filter model

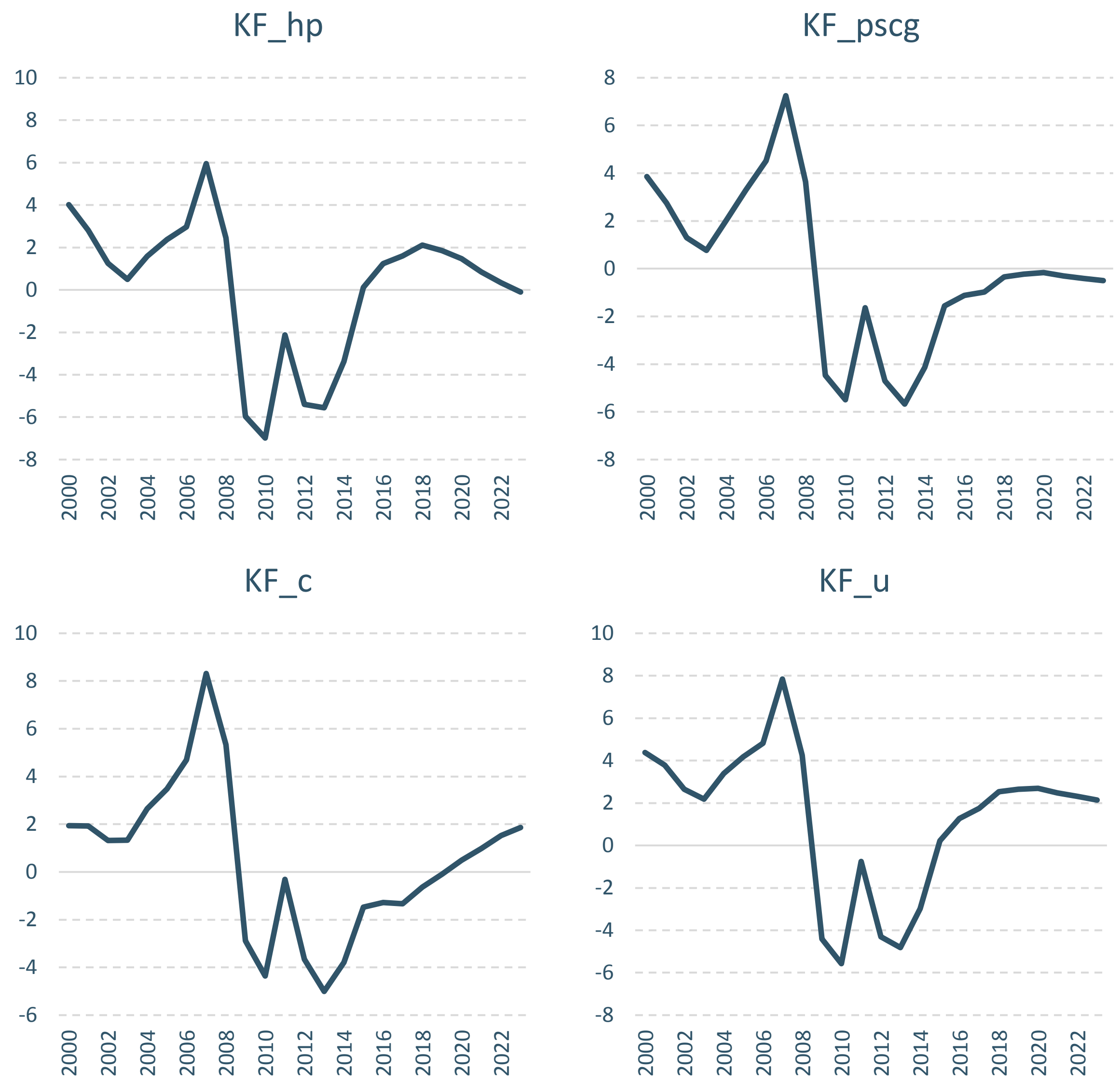
	KF
State cycle equation	
Output Gap (-1)	1.29***
	(0.25)
Output Gap (-2)	-0.46***
	(0.15)
Observations	52
AIC	5.16
Log likelihood	-131.21
Convergence achieved	Yes
Test for Independence	
Box-Ljung Test Statistic	14.876
Critical Value	18.31
P-value	0.14
Test for Homoscedasticity	
Test Statistic	11.60
Critical Value	2.33
P-value	0.00
Normality test	
Test Statistic	69.69
P-value	0.00

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

We now turn our attention to our preferred multivariate kalman filter models. The hypothesised structure for the unobserved trend and cycle components specified for each model assumes the trend component follows a random walk with drift while the cycle component follows an AR(2) process. We estimate the model in a step-wise fashion examining one explanatory variable separately at a time. The variables which were significant and appeared to contain the most relevant information about business cycle fluctuations include house price growth, private sector credit growth, unemployment rate and construction employment as a share of total employment.[17] Each of these variables appear to explain different aspects of the economic cycle when one examines the magnitude of the output gap over time compared with the univariate filter (Figure 4).

[17] Other variables considered included net migration as a share of the labour force, the adjusted current account and PMI (capacity utilisation proxy). They were found to be either insignificant or did not produce historically plausible output gap estimates. Our limited success with the adjusted current account is in contrast to similar model specifications estimated in Casey (2018) and is an issue for further consideration.

Figure 4: Multivariate Kalman filter output gap estimates using domestic GVA



Source: Department of Finance calculations

All four models reach a peak in the region of between 6 and 8 percent in 2007. There are some differences as regards when the output gap reached its subsequent trough but all models suggest a consistent movement towards output gap closure from 2013 onwards.

Interestingly, the model with house price growth and unemployment suggest the economy began to produce above its potential level in 2015. Over the forecast horizon, both models depict a widening of the output until 2018 and 2019. In the case of the house price model, the output gap is projected to close by 2023, while the unemployment model suggests an output gap that remains positive at 2 per cent by 2023. By contrast, the model with private sector credit growth indicates the output gap estimates remain below the line and is marginally negative by 2023. As regards the model with the construction employment, the forecasted rise in this variable results in an output gap estimate that turns positive in 2019 and widens thereafter. These paths are consistent with the implied deviations of the forecasted variables from their long term (i.e. sample) averages.

The relevant estimation output are included in Table 6. All four explanatory variables are found to be significantly related to the economic cycle and exhibit the expected signs.

Table 6: Selected parameters and diagnostics for multivariate Kalman filter model

	KF_hp	KF_pscg	KF_u	KF_c
State cycle equation				
Output Gap (-1)	0.60***	0.67***	0.35**	-0.06
	(0.12)	(0.17)	(0.16)	(0.21)
Output Gap (-2)	-0.19***	-0.53***	-0.55***	-0.71***
	(0.07)	(0.08)	(0.09)	(0.09)
House Prices growth	0.22***			
	(0.05)			
Credit growth		0.23***		
		(0.07)		
Unemployment rate			-1.08**	
			(0.48)	
Construction employment share				3.11***
				(0.71)
Observations	51	32	52	52
AIC	4.85	5.42	4.94	5.04
Log likelihood	-119.74	-82.69	-124.44	-127.09
Convergence achieved	Yes	Yes	Yes	Yes

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

Section 4– Evaluation of output gaps

While some aspects of the model selection process have been outlined above, this section discusses in more detail our approach to assessing the relevance and usefulness of the alternative output gap estimates for fiscal policy.

One of the key challenges to any assessment of output gap estimates relates to the latent nature of the output gap which means there are no outturn data to compare the accuracy of model estimates with. Nevertheless, a range of criteria have been developed which are commonly used to select the best performing methodologies (Hjelm and Jönsson, 2010; McMorrow et al. 2015; Turner et al., 2016; Casey, 2018; Cuerpo, 2018).

The various selection criteria that have been used can be broadly summarised into three categories; economic consistency, statistical robustness and transparency (Cuerpo, 2018).[18] The first criterion relates to the evaluation of macroeconomic variables included in the model and the appropriateness of modelling assumptions that seek to capture economic relationships (e.g. Phillips Curve). It also refers to an assessment of the profile of the output gap that is consistent with accepted business cycle chronology (e.g. amplitude, turning

[18] This discussion draws on the more detailed summary by Hjelm and Jönsson (2010).

points). As regards the statistical dimension, this relates to the evaluation of, for instance, parameter significance, model fit, efforts to address end-point bias, and the stability of real time output gap estimates. In terms of transparency and transferability these are broadly qualitative criteria which focus on data inputs and model replication by external researchers.

The weight placed on individual criteria varies according to the priorities of the organisation and the purpose of the model estimates. For example, central banks may place greater weight on methods which perform better at forecasting inflation. On the other hand, international institutions such as the OECD and the European Commission may be more mindful of factors such as model transparency and cross country model replication. From the Department's perspective, it is appropriate to place greater emphasis on criteria which infer the statistical and economic fit of the estimates, their stability over time, and success in forecasting inflation.[19]

4.1: short-term stability of the estimates

The stability of Ireland's output gap estimates is a particularly relevant issue. Revisions to the estimates of Ireland's output gap using the harmonised methodology are the largest amongst the EU15.[20] Large output gap revisions can undermine the modelling approach as well as fiscal planning through, for instance, associated revisions to cyclically-adjusted budget balance estimates.

Notwithstanding the fact that estimate stability is a desirable characteristic of any methodology, it is not clear what the optimal level of real time stability should be. Real time revisions can, for instance, reflect to some extent the models' efforts to provide better estimates having incorporated revised and/or additional observations. With this in mind, we next outline the measures used to analyse the stability of output gap estimates.

4.2: stability metrics

The first set of measures used to examine the stability of the output gap estimates include the average absolute year-on-year revision and the average absolute initial-final revision. The year-on-year revision corresponds to the average absolute revision to the estimates for all available years between consecutive vintages. The year-on-year revision is calculated as follows:

$$R_{yoy} = \frac{1}{TV}\sum_{t=1}^{T}\sum_{v=1}^{V}\left|OG_{t,v+1} - OG_{t,v}\right| \tag{17}$$

where t denotes the year for which the output gap is estimated and v denotes the vintage. For the initial-final revision, we compare the real time estimate to the final estimate (as per vintage 2016). We calculate the average absolute revision as follows:

$$R_{initial-to-final} = \frac{1}{T}\sum_{t,v=1}^{T}\left|OG_{t,v} - OG_{t,V}\right| \tag{18}$$

It is important to point out that the mean annual revision (MAR) is likely to be positively correlated with the amplitude of the output gap. Therefore, a relatively low MAR may simply

[19] The analysis here applies many of the metrics used in Casey (2018). Casey (2018) also identified other criteria including model complexity which measures the number of code programming operations necessary to estimate a model.
[20] Based on mean absolute revision, see Daly and Murphy (forthcoming).

reflect a lower and (possibly incorrect) output gap amplitude rather than being indicative of a relatively stable model. In an effort to address this issue all preliminary output gap estimates were screened and those with a maximum peak of less than 4 per cent (i.e. indicative of an implausible output gap profile) were excluded.

We next consider the maximum revision and the number of sign changes (on both a year-to-year and initial-to-final basis) as indicators of the performance of the models. One additional check of the real-time estimates is to assess how they perform at cyclical turning points. In this case, we examine model estimates of the output gap based on 2007 output vintage.

4.3: calculation of real time output vintages

Before discussing the results it is necessary to outline the approach taken to construct real time domestic GVA based output gap estimates. Real time vintages for domestic GVA have been published since 2013. However, to extend the number of data vintages back to 1995 we apply the approach taken in Casey (2018) where domestic GVA vintages are assumed to be correlated with real GNP vintage data. The assumed relationship is represented by equation 1. The model parameters are those estimated using ordinary least squares using the GNP and domestic GVA data published in NIE 2017.

Real GDP and GNP data vintages are compiled and analysed in Casey (2018).[21] To conduct the real time analysis GNP forecasts as produced by the Department and published in the respective Budget or Stability Programme Update SPU document are applied to the real time GNP series in order to derive a real time domestic GVA dataset. We do not have access to a readily available dataset of quarterly GDP vintages and therefore are not in a position to examine the stability of the extended HP models' real time estimates.

4.4: analysis of real time revisions

There appears to be notable differences in the year-to-year revisions to the real time output gap across the models (Table 7). The mean absolute revision for the models with house price growth (KF_hp), and private sector credit growth (KF_pscg) are subject to the largest revisions. By contrast the revision for the harmonised approach which we have included as a reference is the smallest.

[21] We would like to thank the author for sharing the data with us.

Table 7: Real time revisions analysis, domestic GVA

	Univariate KF model	Multivariate KF models				CAM[22]
	KF	KF_hp	KF_pscg	KF_u	KF_c	
Year-to-Year Revisions						
MAR	1.24	1.60	1.89	1.36	1.00	0.73
Max Revision	4.0	6.16	3.7	5.6	5.1	4.20
Sign Changes	19	15	2	11	4	19
Initial-Final Revision						
MAR	1.27	2.67	1.78	1.61	2.8	1.98
Max Revision	2.5	5.2	3.4	4.2	9.0	5.50
Sign Changes	4	3	0	3	2	5
2007 Estimate						
Initial	2.4	0.7	7.9	4.1	0.8	-0.70
Final	4.2	5.9	8.0	8.3	9.8	4.80

Source: Department of Finance calculations

In terms of initial-final revisions, the univariate Kalman filter (KF) model has the lowest mean absolute revision while the multivariate Kalman filter with house prices (KF_hp) appears to have the highest value. While a number of multivariate Kalman filter models (KF_hp, KF_c, KF_u) have high revisions relative to the harmonised approach, the absolute value of the output gap is also larger in these models relative to the harmonised approach.

In regards to sign changes, there are notable differences across models. While the harmonised approach (i.e. CAM) estimate shows 19 year-on-year sign changes over the period, it is closely followed by both univariate model KF and the multivariate model including house prices (KF_hp). In contrast the multivariate KF models with private sector credit growth (KF_pscg) and construction as a share of total employment (KF_c) presents with only two and four sign changes respectively. In terms of sign changes over the initial-final horizon all alternative models appear to perform better than the harmonised approach.

In terms of assessing the real-time performance at cyclical turning points, all models suggest an initial positive output gap in 2007 in contrast with the harmonised approach.

4.5: output gap estimates and forecasting inflation

We next turn our attention to the ability of the output gap estimates to explain inflation. This can be viewed as a further means to assess the suitability of the output gap estimates.

The Phillips curve provides the motivation for this assessment. It suggests a relationship between measures of slack in the economy, such as the output gap, unemployment rate and the rate of inflation. However, it is interesting to note that, in the case of Ireland, the open nature of the economy has cast doubt over the relevance of the Phillips curve. Some have argued that inflation is primarily driven by external factors as opposed to the performance of the domestic economy. Indeed, early empirical contributions to this debate find that the relationship between the unemployment rate and inflation is weak or insignificant, suggesting

[22] European Commission's harmonised approach.

that domestic factors play a limited role at best in determining prices (Geary and McCarthy, 1976). This result was attributed to the close links with the UK. Indeed, external factors, such as the sterling exchange rate, have been shown to play a significant role in the determination of inflation (Honohan and Flynn, 1986). However, other papers suggest some evidence for the relationship. Gerlach et al. (2015) use a longer sample than previous studies (covering the period from 1929 to 2012) and find a significant Phillips curve relationship. They attribute past failures to identifying the relationship to the relatively short sample periods analysed. Bermingham et al. (2012) find significant support for the Phillips curve relationship when the short-term unemployment gap is used as an indicator of domestic demand. However, more recently, a number of papers have found evidence of a non-linear Phillips curve in Ireland (Linehan et al. 2017; Byrne and Zekaite, 2018).

Notwithstanding the challenges in identifying such a relationship for Ireland, we test the predictive power of our output gap estimates. As per Casey (2018), we test two Phillips Curve specifications. The first is a backward-looking Phillips Curve. In this specification, agents are assumed to take the inflation rate in the previous period as their expectation for the current period.

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 OG_t \tag{19}$$

In the second specification, agents are assumed for form their expectations based on two factors: the ECB's target inflation rate (of 2 per cent), and the inflation rate in the previous period. The magnitude of the coefficient β_4 captures the relative weight placed on these factors, while β_3 captures the role of expectations in determining the inflation outturn. In other words, this captures the accuracy or the degree of self-fulfilment of inflation expectations.

$$\pi_t = \beta_3 \pi_t^e + \beta_5 OG_t \tag{20}$$

$$\pi_t^e = \beta_4 \pi_{t-1} + (1 - \beta_4) * target \tag{21}$$

The models are run on annual data from 1995 to 2017 and we test three different measures of inflation: CPI, core CPI, and wage inflation. Before discussing the results it is important that to note that inflation has been included in one of the extended HP models while house price growth is included in one of the multivariate kalman filter models. The respective inflation model specifications will be subject to some degree of endogeneity bias.

The Phillips curve regressions using GDP based output gap estimates are shown in Tables 8 and 9. The results show that in the case of the backward looking Phillips curve, the output gap is significant in approximately half of the specifications: the regressions with the HP-filtered output gap and two of the capital models. In terms of goodness of fit, the anchored expectations Phillips curve clearly outperforms the backward-looking specification. Within this framework, the coefficient on the output gap is found to be significant and of the correct sign across all models.

Table 8: Traditional Phillips curve approach, (GDP based OG estimates)

	Capital (HP_K)	Labour (HP_L)	CAM
CPI Inflation			
π_{t-1}	0.58***	0.59***	0.55***
	(0.16)	(0.16)	(0.14)
OG_t	0.19	0.18	0.48***
	(0.11)	(0.11)	(0.15)
Obs	22	22	22
R^2	0.20	0.19	0.41
Core CPI Inflation			
π_{t-1}	0.59***	0.59***	0.55***
	(0.17)	(0.17)	(0.13)
OG_t	0.18	0.19	0.50***
	(0.11)	(0.12)	(0.14)
Obs	20	20	20
R^2	0.28	0.27	0.51
Wage Inflation			
π_{t-1}	0.80***	0.80***	0.79***
	(0.12)	(0.12)	(0.09)
OG_t	0.12	0.12	0.44**
	(0.11)	(0.11)	(0.15)
Obs	18	18	18
R^2	0.63	0.63	0.66

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

Table 9: Traditional Phillips curve with expectations approach, (GDP based OG estimates)

	Capital (HP_K)	Labour (HP_L)	CAM
CPI Inflation			
π_t^e	1.04***	1.01***	0.84***
	(0.19)	(0.20)	(0.19)
OG_t	0.37***	0.31***	0.49***
	(0.10)	0.11	(0.14)
Obs	22	22	22
R^2	0.52	0.45	0.52
Core CPI Inflation			
π_t^e	1.01***	0.99***	0.81***
	(0.20)	(0.20)	(0.16)
OG_t	0.35***	0.33***	0.52***
	(0.11)	(0.11)	(0.12)
Obs	20	20	20
R^2	0.54	0.51	0.61

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

We next test the strength of the relationship between the domestic GVA based output gap estimates and inflation. The results are presented in Table 10 and 11 below and suggest that the alternative estimates of the output gap based on domestic GVA do not have as strong an explanatory power with regard to both price and wage inflation in a traditional Phillips curve setting relative to the output gap estimated by the European Commission's harmonised approach. It is important to note that this is not exactly a surprising finding given that the Commission explicitly use a Phillips curve (New Keynesian Phillips curve) in the estimation of the natural rate of unemployment or NAWRU in the methodology. When using a Phillips curve with expectations and inflation targeting, the output gaps based on domestic GVA do seem to perform better.

Table 10: Traditional Phillips curve approach, (domestic GVA based OG estimates)

	KF	KF_hp	KF_pscg	KF_u	KF_c	CAM
CPI Inflation						
π_{t-1}	0.55***	0.54***	0.51***	0.40**	0.52***	0.52***
	(0.10)	(0.15)	(0.15)	(0.15)	(0.16)	(0.13)
OG_t	0.47*	0.34***	0.36***	0.42***	0.32**	0.54***
	(0.23)	(0.13)	(0.14)	(0.13)	(0.14)	(0.14)
Obs	20	22	22	22	22	22
R^2	0.33	0.32	0.33	0.42	0.27	0.46
Core CPI Inflation						
π_{t-1}	0.54***	0.55***	0.53***	0.37***	0.53***	0.53***
	(0.17)	(0.15)	(0.16)	(0.16)	(0.17)	(0.13)
OG_t	0.47*	0.34**	0.35**	0.44***	0.32*	0.54***
	(0.23)	(0.13)	(0.14)	(0.13)	(0.15)	(0.14)
Obs	20	20	20	20	20	20
R^2	0.33	0.40	0.40	0.51	0.34	0.56
Wage Inflation						
π_{t-1}	0.76***	0.76***	0.76***	0.64***	0.76***	0.74***
	(0.12)	(0.10)	(0.11)	(0.120	(0.12)	(0.09)
OG_t	0.37	0.29***	0.28*	0.38**	0.24	0.47**
	(0.23)	(0.12)	(0.13)	(0.13)	(0.14)	(0.14)
Obs	18	18	18	18	18	18
R^2	0.66	0.71	0.69	0.74	0.34	0.77

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

Table 11: Traditional Phillips curve with expectations approach, (domestic GVA based OG estimates)

	KF	KF_hp	KF_pscg	KF_u	KF_c	CAM
CPI Inflation						
π_t^e	0.91***	0.93***	0.91***	0.73***	0.93***	0.88***
	(0.20)	(0.18)	(0.17)	(0.18)	(0.18)	(0.16)
OG_t	0.61***	0.42***	0.48***	0.45***	0.46***	0.59***
	(0.21)	(0.11)	(0.12)	(0.11)	(0.13)	(0.12)
Obs	22	22	22	22	22	22
R^2	0.45	0.53	0.54	0.58	0.53	0.64
Core CPI Inflation						
π_t^e	0.89***	0.94***	0.93***	0.67***	0.89***	0.88***
	(0.20)	(0.18)	(0.18)	(0.19)	(0.20)	(0.16)
OG_t	0.63***	0.44***	0.48***	0.48***	0.44***	0.59***
	(0.22)	(0.11)	(0.12)	(0.11)	(0.13)	(0.12)
Obs	20	20	20	20	20	20
R^2	0.51	0.61	0.62	0.63	0.54	0.64

Standard errors in parentheses (*** p<0.01; **p<0.05; *p<0.1).
Source: Department of Finance calculations

4.6: composite output gap measures

An alternative to selecting one particular set of model estimates is to combine all estimates with the aim of producing a "central" estimate that is more robust to outliers. This is one approach to addressing the fact that each of the model estimates of the output gap are subject to a margin of error. The upper and lower estimates of the range of model estimates can act as a type of "standard error" bound. The application of such an approach is consistent with the risk management perspective discussed in IMF (2010) as it enables attention to be directed not only at a central forecast but also the error band. The error band would be particularly relevant when various model estimates give very divergent assessments. With this in mind, and following the approach taken by Casey (2018) we compute the mid-range estimates which are calculated as the year-wise average of the maxima and minima of the range of estimates. The model estimates which are included in the calculation of the mid-range measure are listed in Table 12.

Table 12: Composition of mid-range measures

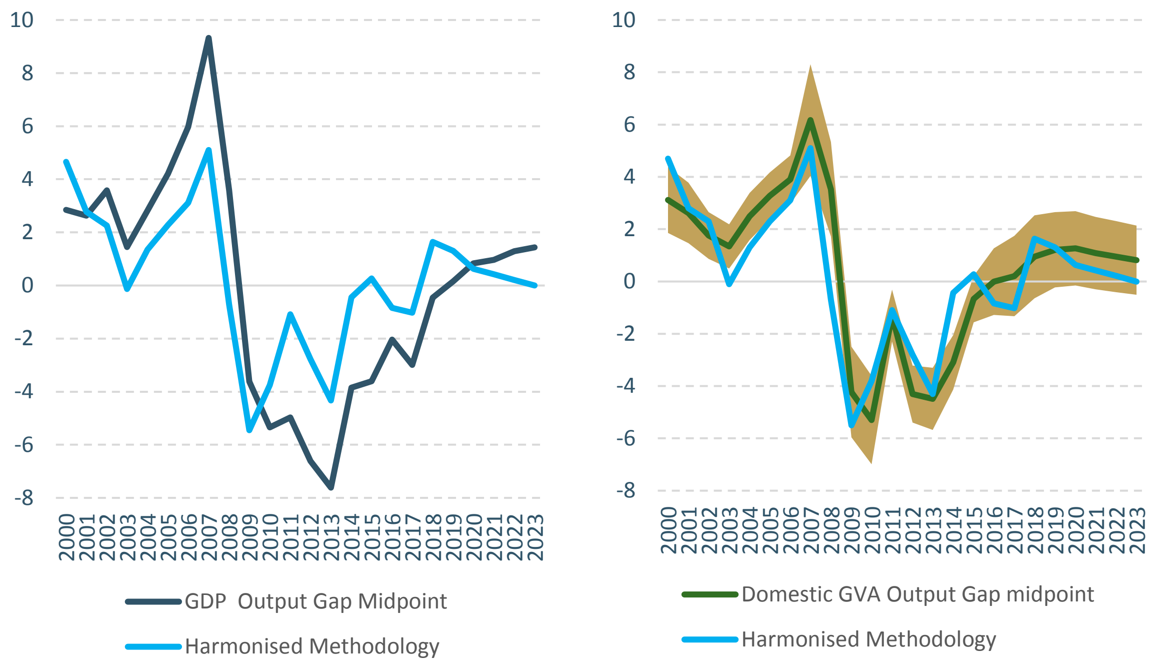
Model	Specification
GDP based models	
1	Extended HP filter: Labour market variables model
2	Extended HP filter: Capital market variables model
Domestic GVA based models	
1	Kalman filter: Trend, random walk with drift; cycle, AR(2)
2	Kalman filter: Trend, random walk with drift; cycle, AR(2); House price growth
3	Kalman filter: Trend, random walk with drift; cycle, AR(2); Private sector credit growth
4	Kalman filter: Trend, random walk with drift; cycle, AR(2); Unemployment rate
5	Kalman filter: Trend, random walk with drift; cycle, AR(2); Construction employment share

Source: Department of Finance.

The mid-point of the GDP and domestic GVA based output gap estimates are presented in Figure 5. Both composite measures suggest that the recovery had taken hold by 2014 as the output gap begins to close. The domestic GVA based estimate turns positive in 2016 peaking in 2018 before marginally closing by 2023. By contrast, the GDP based estimate suggests the output gap closes in 2019 and continues to widen reaching a value of 1.4 per cent by 2023.

The potential for overheating pressures over the forecast horizon suggested by the GDP based output gap estimates are consistent with the forecasted developments in the explanatory variables including unemployment rate, migration and construction employment (and implied deviations from their sample averages). Indeed, developments in these variables reflect the anticipated response to significant under-investment in the residential sector since the financial crisis. House completions are projected to increase over the medium-term and exceed equilibrium demand by the early part of the next decade. From the supply side perspective, such developments may lead to a substantial reallocation of capital and labour from the traded sector to the less productive non-traded sector. In addition, the economy is projected to be operating at full employment from 2020 onwards, this is likely to give rise to upward pressure on domestic prices and wages. The GDP based output gap estimate is the preferred estimate of the Department as it appears to better reflect current indications of some degree of slack in the economy and better captures potential overheating in the future.

Figure 5: Mid-range of GDP and domestic GVA based output gap estimates [23]



Note: The harmonised approach output gap series are the Department's estimates as published in Budget 2019.
Source: Department of Finance calculations.

Section 5– Summary

In any assessment of Ireland's medium-term growth prospects, it must be acknowledged that measurement of potential output is more complex for a small, open economy such as Ireland, which *inter alia* is characterised by significant cross-border mobility of labour and capital. In addition, statistical distortions arising from parts of the multinational sector add an additional layer of complexity to estimating the economy's supply capacity.

Notwithstanding these challenges, this paper has set out to estimate a number of alternative statistical filters including extended HP and Kalman filter models. Within these approaches, we incorporate both financial and labour market variables which are considered to contain important information about Ireland's business cycle fluctuations. Estimates from these models were then assessed taking into account, for example, the plausibility of output gap paths, real time estimate volatility, and the performance of the estimates in explaining inflation.

[23] In analysis of this nature, technical closure rules of the output gap are often applied. We do not impose a closure rule. Instead for the purposes of maintaining consistency with Department's demand side forecasts, we incorporate the Department's actual GDP forecasts, and those for domestic gross value added over the forecast horizon. IFAC (2018) show that the Irish expansionary and recessionary periods typically last for 4.4 years and 2.5 years respectively.

To the extent that a significant amount of uncertainty prevails around each of the output estimates a composite measure was constructed which is based on the mid-range of the selected output gap estimates. The GDP based output gap measure suggests a modest negative output gap in 2018, consistent with limited inflationary pressures in the economy and remaining slack in the labour market. However, the estimated output gap turns slightly positive next year and this widens thereafter, pointing to signs of overheating in the medium term.

These statistical estimates represent an important addition to the Department's approach to assessing the cyclical position of the economy. Moreover, the Department intends to build upon this work in the future with consideration being given to the application of semi structural and structural models. Such models can facilitate more in-depth analysis of future developments in potential output and the output gap.

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Appendix A: The Extended HP Filter

This appendix describes the full range of models tested using the extended HP filter. All models were run to Q4 2023. This sample range was chosen in order to mitigate end-point bias of the estimates for the current period (i.e. 2018). In selecting a model, consideration is given to, first and foremost, the significance of the explanatory values, and secondly, to statistical measures of the goodness of fit, model stability and the plausibility of the results.

Capital Models

The alternative specifications of the capital model tested are shown in Table A1. All specifications include private sector credit growth and core CPI, with the differentiating factor between them being the indicator used to capture developments in the housing sector.

The results show that the first lag of the output gap and core CPI are significant across all specifications and carry the correct sign. Private sector credit growth is significant in all models' with the exception of Alternative 3. Of the four housing sector indicators we consider, three were found to be significant. These include construction sector employment and house price growth. Investment in building and construction was found to be significant only when expressed as a percentage of GDP* (however, this renders private sector credit growth insignificant). Ultimately, Alternative 2 and Alternative 3 were dropped and the selection process focused on the baseline model and Alternative 1, as these are the only models in which all explanatory variables are significant.

Table A1: Extended HP filter model estimates (using relevant financial (capital) variables)

		Baseline	Alternative 1	Alternative 2	Alternative 3
Output Gap	coeff.	0.93*	0.80***	0.75***	0.85***
	t-stat.	(0.20)	(0.38)	(0.40)	(0.34)
Private sector credit growth	coeff.	0.31**	0.27*	0.49***	0.30***
	t-stat.	(0.16)	(0.15)	(0.10)	(0.18)
	lag	5	4	5	5
Construction sector employment	coeff.	1.38***			
	t-stat.	(0.50)			
	lag	0			
House price growth	coeff.		0.50***		
	t-stat.		(0.12)		
	lag		3		
Investment in building and construction	coeff.			-0.10	
	t-stat.			(0.08)	
	lag			1	
Investment in building and construction (share of GDP*)	coeff.				0.21*
	t-stat.				(0.13)
	lag				2
Core CPI	coeff.	1.26**	1.59***	1.94***	2.10***
	t-stat.	(0.52)	(0.39)	(0.36)	(0.32)
	lag	0	0	0	0

Source: Department of Finance calculations

The model estimate stability and statistical measures are shown in Table A2 and the output gap estimates in Figure A1. However, when interpreting these results, it is important to bear in mind the limitations of these criteria. For example, while the assessment of the plausibility of the output gap requires judgement, as previously discussed, one of the key measures of estimate stability, namely the mean annual revision (MAR), is positively correlated with the amplitude of the output gap. Therefore, although a relatively low MAR could be indicative of a relatively robust model, it could also simply reflect a lower, and possibly incorrect, output gap amplitude.

The results show that in comparison to Alternative 1, the baseline model is subject to greater revisions, but performs slightly better in terms of the number of sign changes. For 2007, both models detect a positive output gap in real time, although of different magnitudes. The results for the shortened sample are broadly similar.

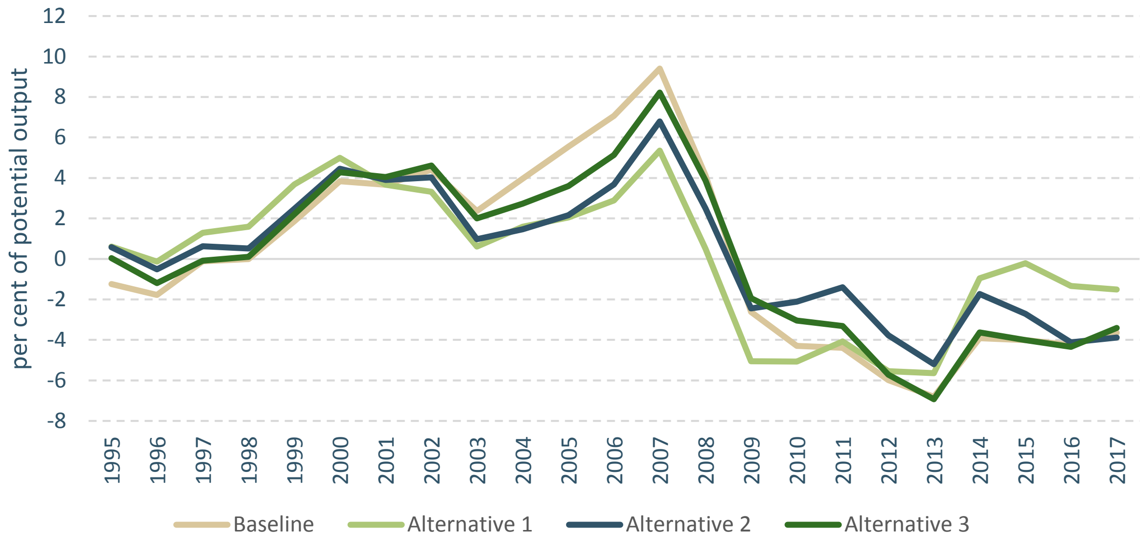
Although the stability and statistical metrics would suggest that, on balance, Alternative 1 is the preferable model, we consider the baseline model as more plausible. The path of the output gap produced by Alternative 1 from 2013 onwards does not appear to be very credible. For example, between 2013 and 2014, the output gap increases sharply by over 4.5 percentage points. Although a narrowing of the output gap would be expected, reflecting the recovery in GDP, a change of such a magnitude appears somewhat excessive. Furthermore, from 2015 to 2017, the output gap is projected to decline, which appears somewhat counter intuitive given the continued strengthening of the economy. These trends appear to reflect the growth of house prices. For example, between 2013 and 2014, growth picked-up sharply by 16 percentage points, whereas between 2015 and 2016, it decelerated by 4 percentage points. However, whilst we are bound to express many of the explanatory variables as growth rates (as opposed to levels) in order to ensure the stationarity of the output gap, there is a potential limitation to this approach. For example, in cases where the level of the variable remains below its long-term trend, but its growth rate exceeds its long-term rate, this approach could produce a misleading signal of overheating, and vice-versa. In the case of house prices, the question of whether price levels have exceeded their trend levels falls outside of the scope of this research. However, growth patterns which diverge from those of levels could potentially give rise to misleading output gap trends. On the other hand, the baseline model, which incorporates the share of employment in the construction sector, produces a smoother output gap closure path, which appears more plausible. On this basis, the baseline model is preferred to Alternative 1.

Table A2: Extended HP filter (capital models); stability measures and model diagnostics

	Baseline	Alternative 1
Year-on-Year Revision		
MAR	0.55	0.33
Max Revision	4.4	2.3
Sign Changes	5	5
Initial-Final Revision		
MAR	3.37	1.88
Max Revision	7.6	4.2
Sign Changes	3	5
2007 Estimate		
Initial	4.0	2.3
Final	9.1	5.2
AIC	4.43	4.26
Log Likelihood	-192.72	-202.08

Source: Department of Finance calculations

Figure A1: Output gap estimates based on extended HP filter capital models



Source: Department of Finance calculations

Labour Models

One alternative specification of the labour model was tested. As shown in Table A3, the alternative model substitutes the unemployment rate for the employment to working age population ratio, while retaining net migration. The results show that all of the explanatory variables, with the exception of the lagged output gap, in both models are significant and carry the correct sign.

Table A3: Extended HP filter model estimates (labour models)

		Baseline	Alternative 1
Output Gap	coeff.	0.97	0.98
	t-stat.	0.03	0.03
Unemployment rate	coeff.	-0.48	
	t-stat.	0.20**	
	lag	0	
Employment to Working Age Population	coeff.		0.45
	t-stat.		0.23*
	lag		0
Net Migration	coeff.	7.61	7.56
	t-stat.	1.51***	1.88***
	lag	0	0

Source: Department of Finance calculations

The stability measures and statistical criteria are shown in Table A4. The revision indicators suggest that the two models perform similarly, and on the basis of the number of sign changes, the baseline models performs slightly better. The estimates for 2007 show that both models detect significant overheating, but the ex-post revision to the alternative estimate is lower. On the basis of the statistical criteria, the baseline model outperforms the alternative.

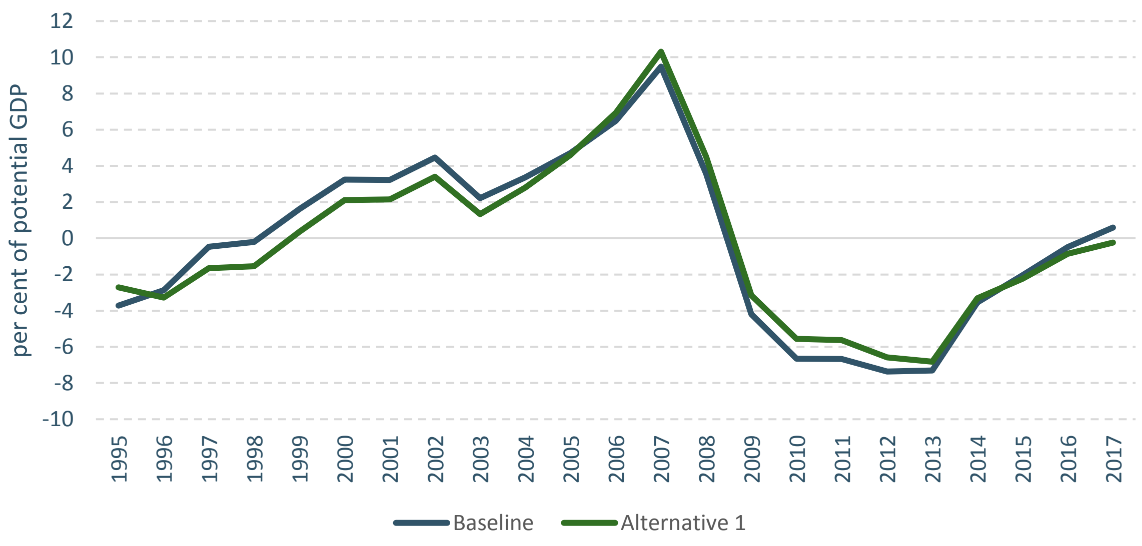
The output gap results are shown in Figure A2. The two sets of estimates are very similar. The baseline model is favoured slightly based on the statistical criteria and stronger significance of the explanatory variables.

Table A4: Extended HP filter labour model estimates

	Baseline	Alternative 1
Year-on-Year Revision		
MAR	0.37	0.53
Max Revision	3.2	5.72
Sign Changes	0	4
Initial-Final Revision		
MAR	1.54	1.04
Max Revision	4.1	4.96
Sign Changes	0	0
2007 Estimate		
Initial	5.8	8.5
Final	9.9	9.9
AIC	4.25	4.44
Log Likelihood	-209.87	-194.08

Source: Department of Finance calculations

Figure A2: Output gap estimates based on extended HP filter labour models



Source: Department of Finance calculations

Stability of GDP based output gap estimates (expanding sample window analysis)

As regards the quarterly GDP series, these vintages are not readily available. Therefore, we are currently not in a position to examine the stability of the extended HP models' real time estimates.

In the case of the GDP based models (i.e. extended HP and univariate kalman filter) we undertake an expanding sample window analysis to assess the stability of the model estimates. More specifically, we take the period 1995 to 2002 as our first sample and then extend the end point forward by four quarters from 2003 to 2016; resulting in 15 sets of output gap estimates. For comparison purposes, the stability measures are calculated based on annualised output gap data.

Stability of GDP based output gap estimates (expanding sample window analysis)

The metrics suggest that, on balance, the two extended HP filter models appear to be more stable in terms of year-on-year revisions and the number of sign changes. The extended HP model with labour market variables outperforms the othergva models on all metrics.

Based on the number of sign changes, the labour model is found to perform best, with no sign changes. Both extended HP models suggest a positive output gap in 2007 based on the reduced sample, although the magnitude of the estimate differs. The revision to this estimate is lowest for the labour model. By contrast, the univariate Kalman filter model indicated that the economy was operating at below capacity in 2007.

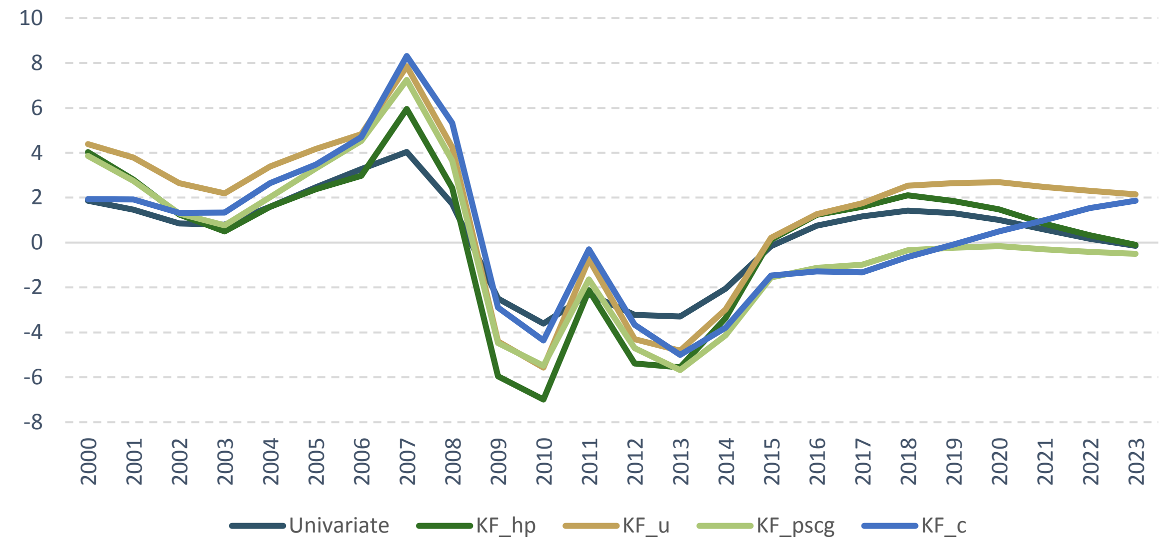
Table A5: Stability of GDP output gap estimates

	Capital (HP_K)	**Labour** (HP_L)	**KF** (KF_GDP)
Year-on-Year Revision			
MAR	0.6	0.3	0.8
Max Revision	4.4	2.2	5.6
Sign Changes	5	0	11
Initial-Final Revision			
MAR	3.4	1.4	5.8
Max Revision	7.6	4.0	12.2
Sign Changes	3.0	0.0	9.0
2007 Estimate			
Sample (1995-2007)	4.0	5.8	-3.0
Final sample (1995-2016)	9.1	9.9	9.2

Source: Department of Finance calculations

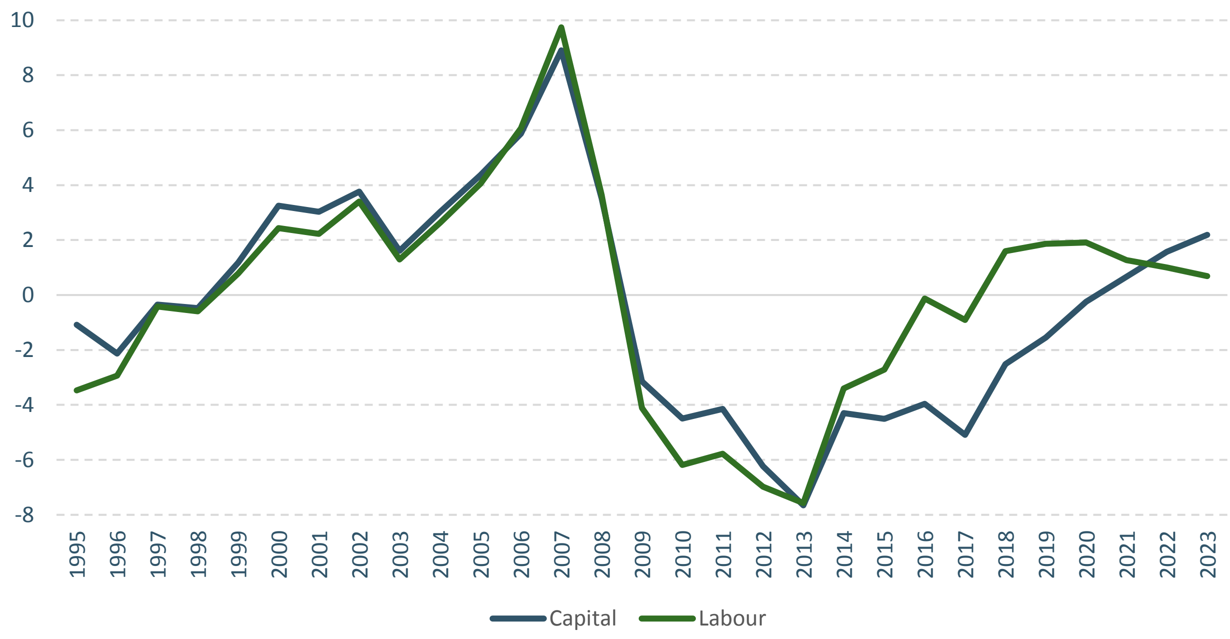
Appendix B: Output Gap Estimates

Figure B1: Domestic GVA based output gap estimates



Source: Department of Finance calculations

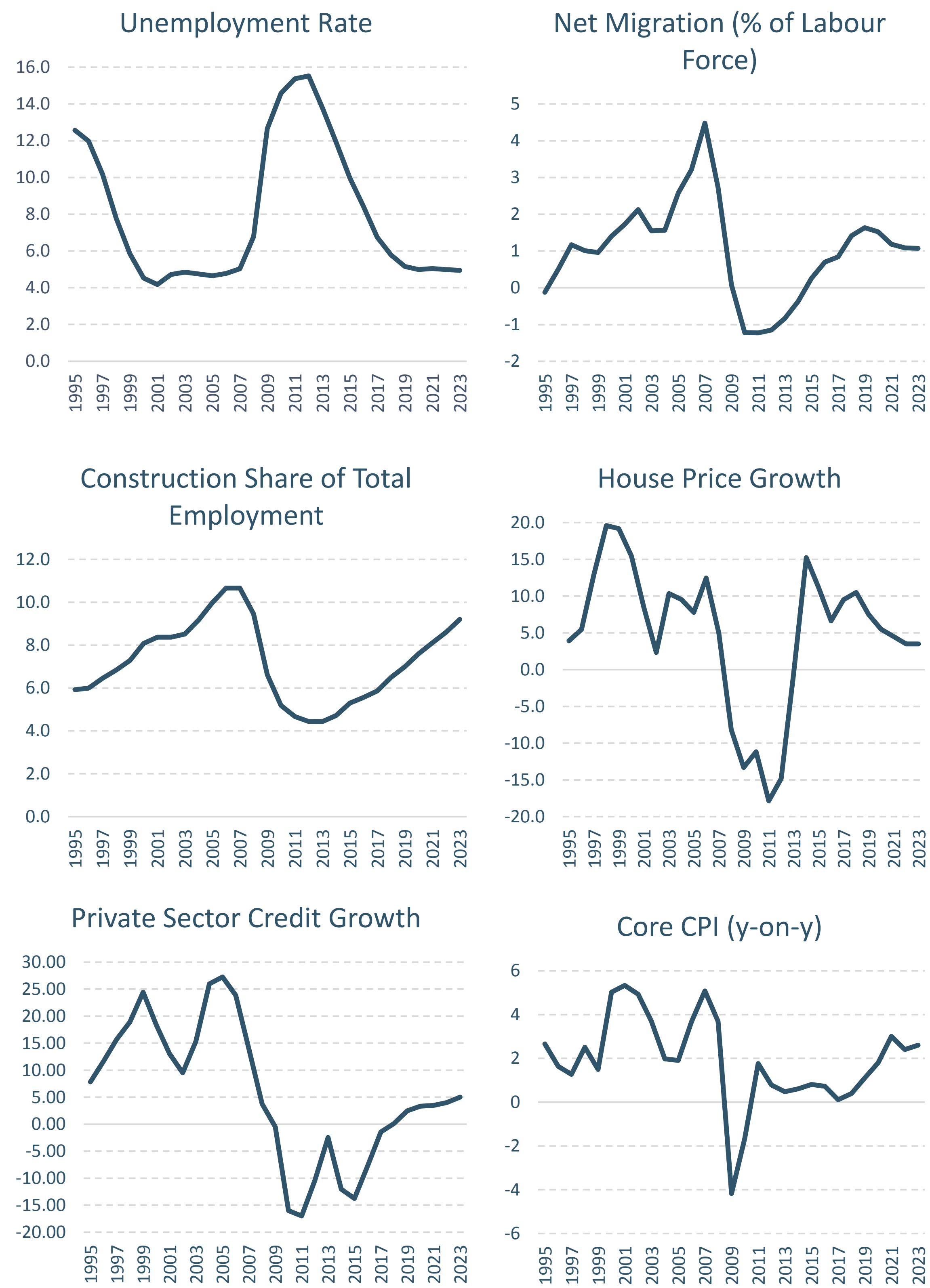
Figure B1: Real GDP based output gap estimates



Source: Department of Finance calculations

Appendix C: Explanatory Variables

Figure C1: Explanatory variables



Source: CSO, CBI, Department of Finance

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