Spending Review 2019

Public Service Employment & Expenditure Modelling

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Report Summary

Background

- Public service exchequer pay at €18.7bn (2019 estimate) represents 31.5% of current expenditure.
- There are two main drivers of the public service pay bill: the number of public servants employed (335,000 Full Time Equivalents Q2 2019) and the rate at which they are paid.
- Since 2014, and the end of the public service recruitment moratorium, the number of public servants employed has grown by 43,755, closing gaps in front line service provision that emerged during the crisis.
- Since 2016, rates of remuneration have also increased as the Financial Emergency Legislation has been progressively unwound.
- In 2018, the latest year of outturn figures, the combined effect of these factors was a 7.3% increase in the exchequer pay bill.
- Pay allocations made through the Budget Estimates reflect anticipated year-on-year changes in public service employment and rates of remuneration. However in-year Government decisions can increase expenditure beyond original Budget Estimates. For example the recruitment of Special Needs Resource Teachers and the implementation of the Public Service Pay Commission recommendations. As a result over the last number of years there has been a level of expenditure growth between Budget Estimate and Outturn.
- No medium term forecast of public service numbers or the Exchequer pay bill currently exists.
- A key recommendation from the 2018 Spending Review is to better “understand the medium-term implications of current pay and employment decisions through better medium-term forecasting of the pay bill”.
- “Improved modelling of medium-term expenditure levels to facilitate expenditure planning, structural reform and delivery, in line with the fiscal rules.” Is also a planned outcome of the Department of Public Expenditure and Reform Statement of Strategy”.
- The aim of this Spending Review paper is to deliver on these commitments, providing improved forecasts, and better forecasting ability.

Methodology

- The paper is based on an analysis of 3 long run time series:
  1. Compensation of Government Employees (CoE) from the National Accounts;
  2. Total Public Service Numbers (including non-Exchequer funded Local Authorities); and
  3. A Rates Index constructed from CoE/Public Service Numbers
- Four models have been devised which examine how these variable have responded to the Economic Cycle and Demographic Change over time.
- Results from these models are used to forecast public service numbers growth to 2025 and expenditure growth to 2022 on a no policy change basis.
- Future work will attempt to improve these models (through inclusion of House Price and Wage data not currently available) and extend the forecast to alternative scenarios, including various potential Brexit outcomes.
Results: Public Service Numbers

- There is a strong positive correlation between overall population growth and public service numbers.
- In examining the composition of population increases, a 1% increase in the age cohort between 15 and 65 years old, will place an additional 3% increase on public service numbers over the long run. A 1% increase in the Old Age Population, will place an additional 1.6% increase in public service numbers over the long run. There is shown to be a marginal impact of the less than 14 years old cohort on public service numbers over the long run, however this has not been the case in more recent years.
- **Model 1**, a time series trend forecast model, conservatively estimates short term numbers growth to reach 342,000 by end 2019 and 350,000 in 2020.
- **Model 2**, a long run cointegration regression model of Numbers and Demography, uses the CSO’s M2F2 population projections, which projects overall population growth of c.10%; over 65 years cohort growth of 33% and under 14 cohort decline of 4% by 2025. **Under this model, in the absence of a policy change which reduces the rate of increase, public service numbers could exceed 410,000 by end 2025 (24% increase on 2018 figures).**

Results: Public Service Pay Expenditure and Rates

- At an aggregate level, the pay bill is positively correlated with the economic cycle.
- 1% change in inflation, has corresponded to a 2% change in the pay bill over the long run.
- 1% change in economic growth (GDP) has corresponded to a 1% change in the pay bill over the long run.
- **Model 3**, an Auto Regressive Distributed Lag (ARDL) Fully Modified Ordinary Least Square (FMOLS) model, uses Department of Finance April SPU macroeconomic, non-shock forecasts and a no policy change assumption. **Under this model Exchequer Pay is forecast to grow by c. 8% per annum, increasing to €24bn by 2022.**
- **Model 4**, an ARDL FMOLS model, provides less conclusive results for rates but does seem to indicate the importance of price inflation as a trigger for wage increases.

Policy Implications

- This work clarifies how the Exchequer pay bill and public service numbers may be expected to evolve over time, based on current public service provision and forecasted changes in the population structure and economy. As such it has broad implications.
- As the analysis of the pay bill is based on the long run relationship between the pay bill and the economic cycle, and the pay bill has historically grown in a pro-cyclical manner, the forecasts can be understood as a continuation of a pro-cyclical pay policy.
- The scale and composition of demographic change will put upwards pressure on public service employment, regardless of the economic cycle.
- Importantly, the forecasts of continued growth in public service employment, and pay expenditure, indicates a level of expansion that may have to be accommodated in Current Expenditure Ceilings under a no policy change scenario.
1. Objectives

In 2019, the exchequer pay bill is estimated to be €18.7bn or approx. 31.5% of gross voted current expenditure\(^1\). After social transfers of approx. €19.7bn this represents the second largest component of current expenditure.

Last year’s Spending Review Series identified the need for improved forecasting of the pay bill over the medium term to improve public expenditure management and for expenditure sustainability. (IGEES, 2018)

“Improved modelling of medium-term expenditure levels to facilitate expenditure planning, structural reform and delivery, in line with fiscal rules” is also a planned positive outcome from the Department of Public Expenditure and Reform, Statement of Strategy 2016-2019 which this paper aims to deliver.

To advance the predictability of the pay bill over the medium term, the first step is to establish the relationship between public service employment numbers, public service wages and the interplay with the economic cycle.

In addition, it is also important to understand how these factors respond and relate to demographic pressures. Analysis conducted in previous Spending Review papers outlines how demographic changes are expected to impact the demand for public services, and by doing so drive increases on related current expenditure. (IGEES, 2016)

The aim of this paper is to build on the analysis published in the 2018 Spending Review series, which examined, at a high level, some of the key issues pertaining to sustainable management of the public service pay bill (IGEES, 2018). Within this scope, the analysis will examine how public service numbers and rates requirements will be affected over the medium term.

Specifically, this analysis adds value to the existing stock of evidence base in three ways:

1. It extends the knowledge of the public service pay bill expenditure, by increasing the stock of data in this area of current expenditure and employment.
2. It builds on existing work, by applying robust econometric analysis to better understand the casual relationship of the pay bill, the economic cycle and demographics.
3. It addresses a key recommendation from the 2018 Spending Review to:

   “Improve predictability [...] to understand the medium-term implications of current pay and employment decisions through better medium-term forecasting of the pay bill”, (IGEES, 2018).

This paper is set out in sections as follows:

- Section 1 outlines the objectives of this analysis.
- Section 2 sets the pay bill in the historical context in order to frame the analysis.
- Section 3 incorporates the recent period of pay bill growth from 2014.
- Section 4 provides a literature review as the theoretical basis on which the econometric modelling is specified and conducted.
- Section 5 presents the data used, highlighting existing limitations where improvements would be desirable.
- Section 6 establishes the methodological approach taken, including the rationale for the models selected.
- Section 7 details the model specification used in the analysis of the pay bill against various independent variables.
- Section 8 summarises the results of this analysis.
- Section 9 briefly outlines some policy implications and next steps.
2. Past Trends and Composition of the Public Service Pay Bill

Current expenditure in Ireland tends to be highly pro-cyclical, a trend which is well established in the literature (Cronin, 2017) (Lane P. B., 2012). International evidence also demonstrate public wage bill spending to be equally pro-cyclical (Lane P., 2003) (World Bank, 2014) (Park, 2015). Irish pay expenditure has followed this trend of pro-cyclicality.

Historically there were large increases in both the number of public servants and the rates of remuneration during periods of economic growth, this was followed by challenging retrenchment in subsequent recessions. This has been characteristic of each decade since the 1970s. For the purposes of this analysis, recent phases of pay policy pro-cyclicality are reviewed to provide a clear narrative of the current position using the most recently available data.

2.2 Pre-Crisis: 2000-2008

Between 2000 and 2008, the Exchequer pay bill more than doubled, rising from €8.0 billion to €17.2 billion, a 115% increase. This was driven by a 73,000 expansion in public service numbers and increases in rates of pay as a result of national wage agreements and other pay awards. The latter of these policy decisions was the primary driver of the Exchequer pay bill increase over this period, accounting for €6.7 billion or 73 per cent.

These substantive increases are highlighted further in Figure 1 below. When graphed with GNI*, it is clear how the rate of increase in the pay bill was greater than the growth in the economy as a whole over the 2000 to 2015 period.

Figure 1: Growth of GNI* and the Pay Bill Net of PRD, 2000 – 2019

Source: DPER Administrative Data and Department of Finance Data.
2.3 Crisis: 2008-2014

From March 2009 to 2014, a moratorium was introduced on recruitment, limited voluntary redundancy schemes, and extraordinary emergency pay legislation which was used to reduce the Exchequer pay bill by approximately €3.7bn. This comprised of €2.1bn reduction in pay as a result of Financial Emergency Measures in the Public Interest (FEMPI) Acts, with the balance arising from a 32,000 reduction in public service numbers.

Notwithstanding the unprecedented level of this retrenchment, the savings only partly reversed the significant increases of previous years. Moreover, both recruitment embargos and emergency legislation have significant drawbacks. Recruitment embargos can deliver quick reductions, taking advantage of the natural churn in the public service workforce, however they are blunt policy instruments that can lead to workforce planning issues and, potentially, a compromised public service delivery.

Pay reductions that cut through contractually agreed terms and conditions require the strongest possible legislation – emergency legislation – the legal basis of which is temporary and is dependent on the existence of a financial emergency. They can also negatively impact on motivation of the workforce and therefore can have implications for service performance. This illustrates the difficulty in reducing expenditure on pay once additional public service numbers and/or higher rates of remuneration are approved.
3. Current Trends and Composition of the Public Service Pay Bill

3.1 Post Crisis: 2014-2019

Since 2014, as the economy has recovered from the last economic shock, the pay bill has grown quickly. In gross terms, the pay bill has increased from €14.7bn to €18.7bn, an increase of €4bn billion or 21% between Budget 2014 and Budget 2019.

For the period 2014-2018, where outturn figures are available, the increase was €3bn with numbers adding approximately €2bn and rates €1bn. Growth in public service employment in the aftermath of the crisis was to be expected as gaps that emerged due to the recruitment embargo were filled and services expanded to cater for a growing population. Later sections of this paper will examine how these trends may be expected to continue into the future.

Figure 2 shows the year-on-year growth rates in the pay bill, disaggregated by numbers and rates. Of note is the constrained growth in rates up to 2017, and the stronger growth since. When combined with the consistently large increases in public service numbers, this has resulted in overall pay bill growth 7.4% in 2018.

Figure 2: Decomposition of the Pay Bill, 2000-2018

Source: DPER administrative Data

Pay allocations made through the Budget Estimates reflect anticipated year-on-year changes in public service employment and rates of remuneration. However in-year Government decisions can increase expenditure beyond original Budget Estimates. For example the recruitment of Special Needs Resource Teachers and the implementation of the Public Service Pay Commission recommendations.
As a result over the last number of years there has been a level of expenditure growth between Budget Estimate and Outturn.

### 3.2 Numbers Pressures

Public service numbers reached a trough in Q4 2013. Since then the public service has expanded rapidly with a marked compositional shift to Education and Health.

End Year 2017 figures indicate that exchequer funded public service staffing levels (excluding Local Authority staff) have exceeded the pre-crash total.

End year 2018 figures indicate that total public service numbers (including Local Authority Staff) have exceeded the pre-crash total numbers, despite nearly 9,000 fewer staff in Local Authorities (non-Exchequer funded) than peak.

The increases in public service numbers reflect Government policy prioritisation: 90% of the increase has been in Education and Health. This has led to a compositional shift in the numbers, with some areas still seeing reductions public service numbers over this recovery phase, but being offset by large increases in other demand led service areas. When the transfers in Departmental responsibilities are accounted for, Health and Education have exceeded pre-crisis staffing levels.

**Figure 3: Increases in Public Service Numbers since signing PSSA, Q4 2016 – Q2 2019**

<table>
<thead>
<tr>
<th>Public Service Numbers</th>
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<tbody>
<tr>
<td>Defence</td>
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<tr>
<td>Employment Affairs &amp; Social Protection</td>
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<td>Community and Rural Affairs</td>
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<td>Taoiseach’s</td>
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<td>Culture, Heritage and the Gaeltacht</td>
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<td>Communications, Climate Action &amp; Environment</td>
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<td>Business, Enterprise &amp; Innovation</td>
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<td>Rural &amp; Community Development</td>
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<td>Transport</td>
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<td>Agriculture</td>
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<td>Children &amp; Youth Affairs</td>
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<td>Finance</td>
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<td>Foreign Affairs</td>
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<td>Public Expenditure &amp; Reform</td>
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<td>Justice</td>
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<td>Housing, Planning &amp; Local Government</td>
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<tr>
<td>Education</td>
</tr>
<tr>
<td>Health</td>
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<td>Public Service Numbers</td>
</tr>
</tbody>
</table>

**Source:** DPER administrative Data

Table 1 below shows the scale of increases in public service numbers from 2016 to 2019 and the extent to which this has been driven by un-forecasted in year numbers growth.
### Table 1: Public Service Numbers Budget Estimate to Outturn 2016-2019

<table>
<thead>
<tr>
<th></th>
<th>Budget Estimate</th>
<th>Outturn</th>
<th>YoY Outturn Numbers Growth</th>
<th>In-year Numbers Growth above Budget Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016</td>
<td>304,177</td>
<td>309,751</td>
<td>8,163</td>
<td>5,574</td>
</tr>
<tr>
<td>2017</td>
<td>314,160</td>
<td>320,758</td>
<td>11,007</td>
<td>6,598</td>
</tr>
<tr>
<td>2018</td>
<td>325,873</td>
<td>328,921</td>
<td>8,163</td>
<td>3,048</td>
</tr>
<tr>
<td>2019</td>
<td>336,158</td>
<td>335,594*</td>
<td>6,673</td>
<td></td>
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<tr>
<td>Average</td>
<td></td>
<td></td>
<td>8,502</td>
<td>5,073</td>
</tr>
</tbody>
</table>

*Q2 2019

### 3.3 Rates Pressure

The Lansdowne Road Agreement 2016-2018 (LRA) commenced the process of unwinding the Financial Emergency Legislation. Threatened strike action on behalf of an Garda Siochana resulted in a Labour Court Recommendation in November 2016 costing approximately €50m. In turn, to ensure continued adherence to the collective agreement, the Government agreed to bring forward a €1,000 payment to those earning less than €65,000 from September 2017 to April 2017 at a cost of €120m and the early negotiation of a successor, the Public Service Stability Agreement 2018-2020 (PSSA).

Consequently, in 2018, there were two pay agreements running concurrently and the unit cost of public servants increased by 4.2%. Demographic changes within the public service may have contributed to this overall figure through incremental progression.

Rates pressures have continued to manifest during the lifetime of the PSSA.

Firstly it should be noted that all additional public servants detailed in the previous section are employed at rates that increase under the PSSA. In addition the compositional shift towards the single scheme has increased the cost of the reduced rates of Additional Superannuation Contribution payable by Single Scheme members as part of the agreement.

Within the Public Service Stability Agreement, the examination of recruitment and retention by the Public Service Pay Commission has resulted in costed recommendations of approx. €30m in Health and Defence.

Dealing with legacy salary scale issues relating to new entrants, in line with the commitment in the Agreement and amendment to the Public Service Pay and Pensions Act 2017, has brought approx. €78m into the life time of the Agreement.
Further expenditure pressures have come from Trade Unions, particularly in the Health Sector, who have gone on strike to secure pay increases beyond the basic terms of the Public Service Stability Agreement. This can be seen in the days lost to strike action in 2019, with the CSO reporting a total of 27,415 days lost to strikes in Q1 2019 compared to 290 in Q1 2018. The majority of these have been in the Public Service. Labour Court Recommendations for the resolution of these disputes will also be cost increasing above the PSSA baseline.

All these expenditure pressures will require the diversion of additional resources to pay, beyond that which was planned for within pre-committed expenditure under multi annual ceilings.

### 3.4 Current Expenditure Projections

A complete multi annual projection for the pay bill does not exist. Public service pay is however one of the principal components of current expenditure and medium term ceilings for current expenditure are regularly published, most recently in the Mid-Year Expenditure Report 2019² (MYER).

Table 2 below presents the 2019-2024 current expenditure ceilings from the MYER in the context of the recent evolution of current expenditure and pay expenditure.

The first thing to note is that growth in the public service pay bill accounted for 43% of total current expenditure growth between 2015 and 2019. This shows the extent to which pay has been prioritised over other forms of current expenditure in recent years. For example, in 2018, the last year with outturn figures, total gross voted current expenditure growth grew by 5.6% year on year, however the pay bill grew by 7.3% year on year. As stated in section 3.1 and previously detailed in the 2018 IGEES paper, a level of catch up growth was to be expected emerging for the crisis, the focus of this paper is how the pay bill may be expected to grow over time with reference to the economic cycle and demographics.

<table>
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<tbody>
<tr>
<td><strong>Gross Voted Current</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>€52</td>
<td>€54</td>
<td>€57</td>
<td>€59</td>
<td>€60.7</td>
<td>€62.7</td>
<td>€64.8</td>
<td>€66.9</td>
<td>€69</td>
<td></td>
</tr>
<tr>
<td><strong>% Change</strong></td>
<td>1.80%</td>
<td>4.3%</td>
<td>5.6%</td>
<td>3.9%</td>
<td>2.4%</td>
<td>3.2%</td>
<td>3.3%</td>
<td>3.3%</td>
<td>3.3%</td>
</tr>
<tr>
<td><strong>Gross Voted Pay</strong></td>
<td>€15.6</td>
<td>€16.5</td>
<td>€17.7</td>
<td>€18.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>% Change</strong></td>
<td>3%</td>
<td>6%</td>
<td>7.3%</td>
<td>5.6%</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</table>

² Mid-Year Expenditure Report 2019, July 2019
Table 2 implies that public service pay has taken a greater share of current expenditure over time. This can be graphically illustrated in Figure 4 which shows that, since the low in 2014, public service pay as a percentage of current expenditure has grown from 27% in 2014 to 30% in 2018, returning to a pre-crisis level seen in 2007.

**Figure 4: Pay Bill (€bn) as % Share of Gross Voted Current Expenditure**

![Graph showing the pay bill as a percentage of current expenditure from 2007 to 2018. The graph illustrates a trend where the pay bill has increased from 27% in 2014 to 30% in 2018, returning to a pre-crisis level seen in 2007.]

*Source: DPER administrative Data*

The second point to note from the Mid-Year Expenditure Report is that the published medium term current expenditure ceilings imply a considerable lessening in the rate of current expenditure growth. By extension this implies a moderation in pay expenditure growth, unless pay is continued to be prioritised over other forms of current expenditure beyond a level to be expected from catch up growth.

Subsequent sections of this paper will explore the relationship between the pay bill and both the economic cycle and demographic change to forecast how the pay bill might be expected to move in different growth scenarios.
4. Theoretical Framework of Analysis

The theoretical framework for conducting this analysis is based on both international and domestic empirical analysis of public service pay bill expenditure. The framework proposed in this paper is represented below:

Figure 5: Proposed Theoretical Framework of Analysis

4.1 Macroeconomy & Political Economy

Macroeconomy

The literature identifies the distinction between the ‘institutional’ and ‘functionalist’ determinants of the public sector wage bill (Park, 2015).

The institutionalist literature argues that, given the lack of market discipline, the nature of the wage setting institutions of a country will determine the public wage growth or application of moderation. However, the ‘functionalist’ explanation of the pay bill argues that the public service pay bill, as with other areas of current expenditure is a function of the fiscal position of the economy.

For example, Park (2015) examines a large literature of case studies detailing economic shocks experienced by countries over the last fifty years, including the more recent experience of EMU countries. This literature suggests that the impetus for public sector wage moderation can lie in factors such as public fiscal imbalances and economic shock, which generate a significant pressure for governments to tighten public sector wages, at least for a defined period of time.

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3 See full list of references at end of document.
In recent decades Ireland has experienced two significant periods of adjustment in fiscal policy. Irish fiscal consolidation from 2008-2014 (IGEES, 2017), and during several budgets during the 1980’s recessionary period (Hardiman, 2014). There is significant empirical evidence which details the highly pro-cyclical tendency of Irish current expenditure (Cronin, 2017) (Lane P. B., 2012), as well as capital expenditure to an even greater extent, (Oyewole, 2019).

Given the scale and significance of these two periods of adjustment in Irish fiscal policy, alongside the pro-cyclicality of Irish public expenditure, it can be argued that Ireland aligns closely to the category of ‘functionalist’ as the explanatory power of the pay bill.

Central to this analysis is to examine the lagged \( (t-1) \) effect of macroeconomic performance, and movements in pay bill expenditure, with measures to include GDP and GNI* alongside Revenue data used.

The importance of budgets are identified in the literature, however due to econometric and data issues, modelling these variables would be mutually exclusive. In simpler terms, Revenue is a function of output, and therefore using this variable in the same regression as GDP or GNI* unnecessarily increases the variable inflation factor, thus reducing the models explanatory power. GDP and GNI* are also modelled separately.

**Political Economy**

Identified in the literature as another important determinant of the public service wage bill, are factors of the political economy that exists (Lane P., 2003). These factors can take the form of ‘multiple power blocs’ who compete for a share of fiscal revenues, and they can be internal – Trade Unions, or line Departments.

Lane (2003) argues that fiscal competition increases during upturns, thus this could increase spending disproportionately relative to the increase in economic growth and revenue. Therefore, the share of pay bill expenditure of total revenue is considered in this analysis.

The World Bank (2014), also notes that there is cross country evidence of stronger wage bill growth in pre-election periods. This has been incorporated into the regression analysis through the use of an election dummy variable.

**4.2 Numbers**

**Population & Demographics: Expenditure Requirement**

Previous analysis and work in this area have highlighted the important role that demographics play in public service numbers and the public finances. IGEES work conducted (2016) outlines how evolving
demographics will place additional pressure on the public expenditure over the medium term, particularly in the areas of health and pensions, related to the effects of the aging population and increases in mortality rates. ESRI (2017) research also identifies how increases in older age cohorts and population growth as a whole, will impact health care demand in the long term.

Demographics: Funding Capacity
The Department of Finance (2018) highlighted the risks to future potential output and revenue as our population matures. In short, the analysis predicted an increase in the requirement to deliver public services, with simultaneous reductions in the capacity to do so. The effects of overall population growth, and the changes in age profile cohorts are accounted for using CSO population data and DPER public service numbers in the whole time equivalences (WTE).

4.3 Rates
Cost of Living
Another important determinant of public sector wages, and one that is established in the literature, is the cost of living (Mayock, 2016) (Blanchard, 1986) (Bover, 1989) (Winters J., 2009).

(i) General Prices

The interaction between prices and wages is well established in economic theory. The spill-over effects of nominal adjustments of wages and prices across labour and product markets are commonly known as the wage–price spiral. An increase in aggregate demand prompts firms to adjust nominal prices upward in order to maintain or increase the mark up of prices over wages. Concurrently, workers attempt to adjust nominal wages upward, in order to maintain or increase real wages levels. After a decrease in aggregate demand, the process of adjustment of nominal wages and prices are consistent with a simultaneous reduction in profit mark ups and real wages. Wage–price setting may be instantaneous or staggered, allowing for a slow process of nominal adjustment (Kandil, 2007).

The analysis will use consumer prices measured using the Central Statistics Office Consumer Price index.

(ii) House Prices

An important feature of employee compensation is the cost of living, therefore by extension the issue of housing / rent is considered in the literature, the theory is that workers will seek to be compensated for higher housing and rent expenses. (Bover, 1989), (Winters J., 2009) and (Mayock, 2016).
Labour Market Activity

Given its composition of total employment within the economy, developments in public sector recruitment and wages are likely to produce significant spill over effects to the labour market, (Caponi, 2017). The labour market is made up broadly of labour supply and labour demand, and the interaction between employers and employees. As the public administration is effectively the employer of public service employees, the wider labour market is modelled. Important to note here, is that a broad definition of the labour market is chosen in this analysis, rather than modelling unemployment alone. This is because the standard definition of the unemployment rate may be too narrow a variable, especially when the unemployment rate falls to low levels. Thinking about underemployment and broader measures of labour supply is important when considering the slack, or lack thereof, in the labour market.

Therefore, a seasonally adjusted Live Register variable is included to capture the wider labour market. The Live Register is used to provide a monthly series of the numbers of people (with some exceptions) registering for Jobseekers Benefit (JB) or Jobseekers Allowance (JA) or for various other statutory entitlements at local offices of the Department of Social Protection.

Across Europe, compensation of government employees is normally higher than in the private sector economy. This analysis shows that private sector wages are more sensitive to public wage increases than vice versa (European Commission, 2014). Although there is two-way causality interlinkage in the relationship dynamic, there is some evidence that public wages may be sensitive to private wages, (Ana Lamo, 2013) and (Afonso, 2014).
5. Data and Transformations

5.1 Data

Dependent Variables (DV)

The econometric analysis is presented with three separate dependent variable regression models. Firstly, the dependent variable is the Number of Public Servants, using data from the Department of Public Expenditure. Secondly, the dependent variable used is ‘Rates’, which is calculated by dividing the number of public servants by the pay bill. Thirdly, the CSO’s compensation of government employee’s expenditure figure is used from the national accounts. It is important to note the distinction of the broad definition of the public service pay bill and its employees, in relation to the exchequer funded pay bill.

(i) Compensation of Public Service Employees

The broad definition of the public service pay bill, is the CSO’s Compensation of Employees data. This represents the pay bill of all public servants, including Local Authority employees, and includes an imputed pension contribution. This figure is the most reliable in terms of robustness and consistency over the time period and is taken directly from the national statistics agencies main database, under Government Finance Statistics, ESA 2010 Code (D1).

The reason for choosing the CSO’s compensation of public service employees, as the pay bill indicator, is due to data quality and communication. This is an established indicator, produced by the national statistics agency, with a time series of over 20 years. This also incorporates the most general understanding of the public pay bill, which includes state bodies, and local authorities.

In comparison, the exchequer funded figure was only disaggregated between pay and pension’s post 2011, and a longer time series back to 1997 is an estimate. This figure also does not include local authorities, as these are not exchequer funded, however the general public may not be aware of this distinction. The CSO figure also amalgamates historical re-classifications of public service employees over time, such as the re-classification of health board employees from local authorities to HSE exchequer funded employees. This usefully avoid a break in the series being examined.

Although the levels of the variables differ, they follow the same deterministic trend, shown in figure 6 below. This is examined more formally in table 3 below, which shows the three variables are almost perfectly correlated with each other, with a correlation coefficient above 99% at the 1% confidence interval.

4https://www.cso.ie/px/pxeirestat/Statire/SelectVarVal/Define.asp?maintable=GFQ01&PLanguage=0
Table 3: Pearson Correlation Coefficient between various Pay Indicators

<table>
<thead>
<tr>
<th>Pearson Correlation of Pay Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation of Employees</td>
<td>1</td>
</tr>
<tr>
<td>Exchequer Pay &amp; Pensions</td>
<td>0.995*** 1</td>
</tr>
<tr>
<td>Exchequer Pay</td>
<td>0.996*** 0.995*** 1</td>
</tr>
</tbody>
</table>

*** = significance at 1% level.

Figure 6: Pearson Correlation Coefficient between various Pay Indicators

Source: DPER administrative Data and CSO compensation of employees.

(ii) Public Service Numbers

Again, the Dependant Variable uses the broad definition of public service numbers, as opposed to exchequer funded numbers, to relate directly to the broader definition of the public service pay bill, the CSO’s compensation of employees.

(iii) Public Service Rates at Unit Labour Cost

The Rates dependent variable is derived from the first two dependent variables, and as such represents a unit labour cost (ULC).

\[
Real\_Rates_{t1} = ((CoE_{t1} \div Numbers_{t1}) \times 1,000) - \pi
\]
Independent Variables (IV)

The dependent variables are then examined in relation to a number of independent variables which have been identified in the literature.

5.2 Limited Observations

In trying to establish a long run relationship between the variables, it is important to allow for a long enough time horizon for various equilibriums to be observed and short term deviations of shocks to revert to their means. This is why the time period of 1980-2018 was chosen. Shorter time periods of 2000-2018 were considered, however more cycles of the data was deemed more appropriate for a causal analysis.

As such, raw annual data of only 38 observations was at the low end of sufficient observation given the time series is from 1980-2018. Given the use of lagged variables, this reduces the observations to just 37. A key contribution of this analysis is to increase the long run time series of various national data by combining both the historical annual data, with the more recent national quarterly data.

To do this, data was interpolated as a means to increase the annual data from 1980-2018 into a quarterly time series, and subsequently combine with the current national quarterly data from 2000 Q1 – 2018 Q2.

5.3 Data Transformations

An issue commonly encountered with time series data, is that the raw data must be transformed before processing in order to be ready to perform analysis. There are many reasons for this across different types of datasets. The data and econometric issues have been identified and documented as well as the adjustments required and the transformation techniques used on the data in this analysis.

Interpolation of Annual Data combined with Quarterly Data.

Interpolation is a mathematical technique that is used estimate a value of (a function or series) between two known values. Running a quadratic interpolation function using EViews software, quarterly data was derived by using known annual data, with a high degree of accuracy.
Once the derived data was constructed, this was compared to actual quarterly data to examine its accuracy. A sample is shown below. From the sample below one can see the data is accurate to -0.1% on average. Across the entire sample, the data is accurate to -0.05%.

Table 4: Excerpt of Interpolation of Annual GDP to Quarterly 1994 Q4 – 1995 Q4,

<table>
<thead>
<tr>
<th>GDP CSO (Current SA) (actual)</th>
<th>GDP Interpolated from Annual Series</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994Q4</td>
<td>12,582,001</td>
<td></td>
</tr>
<tr>
<td>1995Q1</td>
<td>13,186,000</td>
<td>-0.32%</td>
</tr>
<tr>
<td>1995Q2</td>
<td>13,626,000</td>
<td>-0.71%</td>
</tr>
<tr>
<td>1995Q3</td>
<td>13,899,000</td>
<td>0.00%</td>
</tr>
<tr>
<td>1995Q4</td>
<td>14,121,000</td>
<td>0.92%</td>
</tr>
</tbody>
</table>

Source: Authors own calculations of CSO data. Average accuracy rating 99%. Average variance -0.05%.

Stationarity

When estimating time series models, it is a requirement that the data is stationary. A simple definition of a stationary time series is data that has a constant mean and variance over time. A stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times.
However, economic time series datasets are often non-stationary – that is, they do not have a constant mean and variance over time. When two non-stationary series are regressed on one another, it creates the risk that results will be obtained, but that these results may be spurious. Spurious results are results that are not robust and may result from underlying trends in data rather than a real causal relationship. Therefore, the testing for, identification and correction of non-stationary series is a critical part of time series analysis. In Section 5 a more detailed procedure for both testing and treating stationarity is outlined.

**Logarithmic Transformations**

The first transformation is the introduction of logged versions for all appropriate continuous variables. Many of the data used in this analysis exhibit a strong trend. In such cases, the trend may dominate other elements of the time series, and therefore obscure its relationship to other variables. Taking the log form of the variables helps to linearize the exponential trend, (Asteriou, 2006). Also, by taking the variable in log form, an elasticity analysis is provided for. This is useful for interpretation of results from the various regression analysis shown as correlation coefficients.

**Differencing**

The second stage of specification deals with stationarity in the variables. Differencing of data is performed on the across all variables in this analysis. The aim in time series is to transform the data to ensure a stationary stochastic process, this is discussed in detail in section 4.3 below.

**Seasonal Adjustments**

Seasonal adjustment is a statistical method for removing the seasonal component of a time series that exhibits a seasonal pattern. This is a common technique in time series when requiring to analyse the trend, and cyclical deviations from trend, of a time series independently of the seasonal components.

Some of the raw quarterly data have seasonal cycles which are not adjusted for by the CSO, including Revenue receipts and Compensation of public service employees. It was necessary to adjust for this component in order to understand what the underlying trends are with the seasonal patterns in the data excluded.

**Centred Moving Average Method**

To calculate a moving average, one averages consecutive groups of observations in a series. The centred moving average technique, is the average of two 12-month averages that are offset by 1 month relative to each other. Shown below is an example of how the CSO’s compensation of public employee’s data was ‘smoothened’ using this method.
Deflationary Adjustments: Nominal vs Real

As most of this data is denominated in monetary values, nominal series therefore incorporate a price component that can obscure fundamental aspects of the data. As such it is necessary to adjust nominal data to real data by deflating by consumer price increases in the same period.

For example, as shown below, in the early 1980’s when inflation was between 15% and 20% per annum, growth rates in public service wages was significantly diminished by higher consumer prices. As such it was appropriate to reduce growth, to their real value.
5.4 Testing and Transforming for Stationarity

Unit Roots and Spurious Regressions

As outlined in Section 4.2, there are important differences between stationary and non-stationary time series data, of which must be controlled for in this analysis.

Unit root tests are used to detect the presence and form of non-stationarity. In order to perform time series analysis, the data must be transformed to a stationary form. If a time series is stationary, its mean, variance and autocovariance (at various lags) remain the same no matter at what point in time they are measured, they are time invariant. If the data has a trend element, a form of trend removal is also required, which can be conducted by first differencing or time-trend regression. First differencing is more appropriate for $I(1)$ time series while the time-trend regression technique is more appropriate for trend stationary $I(0)$ time series.

A non-stationary time series is considered integrated if it can be transformed by first differencing once (or a very few times) into stationary data. The order of integration is the minimum number of times the series needs to be first differenced to yield a stationary series. An integrated of order 1 time series is denoted by $I(1)$, while a stationary time series is said to be integrated of order zero, $I(0)$.

Unit root tests are used to determine which technique should be used on the trending in order to render the data stationary and to enable analysis to be performed.

Unit Root Testing

When it comes to testing for stationarity, three tests were considered. The first is a graphical analysis, the second is the Augmented Dickey-Fuller test, and the third the Phillips-Perron test.

Graphing a variable over time gives a good sense of whether it is stationary or not, however, most statistical packages provide tests of stationary, the most common of which are the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) Unit Root test. The computed statistics are then compared to critical values (produced by the test). If a tested variable is non-stationary, the appropriate transformation is to convert it to stationary form. Such transformations include first differencing ($\Delta = ot + t - 1 x \psi x$) and the application of filters and smoothers, the latter depending on whether a variable exhibits a seasonal trend.

A graphical analysis for stationarity can be used to assess whether the data fluctuates around a constant mean and possesses a common variance. Correcting for stationarity, firstly it is required to identify the order to which the data is integrated. In simple terms, if data is non-stationary in levels but a conversion to first differences (change in the variable) yields a stationary variable, then the
variable can be deemed integrated of order one $I(1)$. If the variable is non-stationary in levels and first differences but is stationary in second differences, then the data can be deemed integrated of order two $I(2)$ and so on.

Most time series data in economics are integrated of order one. Correcting for stationarity involves applying differences until the data becomes stationary. Normally, variables that are non-stationary should not be used in an estimation procedure. An exception to this is the special case of cointegration. The key requirement for a cointegration model is that the variables are integrated of the same order.

After examining the data graphically, the null hypotheses of an autoregressive unit root series was tested using the Augmented Dickey Fuller and Phillips-Perron tests.

**Augmented Dickey Fuller Test**

One of the most common tests for unit roots is the Dickey-Fuller test (1979) which sets the null hypothesis that $Y_t$ is non-stationary (random walk without drift) and in the alternative hypothesis, $Y_t$ is a stationary AR(1) process. The Dickey–Fuller test involves fitting the regression model by ordinary least squares (OLS). The augmented Dickey–Fuller test’s regression includes lags of the first differences of $Y_t$ which will account for serial correlation.

**Phillip Perron Test**

The Phillips– Perron (PP) (1988) test involves fitting the regression model $\Delta y_t = \rho y_{t-1} + (constant, time trend) + u_t$, and the results are used to calculate the test statistics. The PP tests correct for any serial correlation and heteroscedasticity in the errors $U_t$ non-parametrically by modifying the Dickey Fuller test statistics. Phillips and Perron’s test statistics can be viewed as Dickey–Fuller statistics that have been made robust to serial correlation by using the Newey–West (1987) heteroscedasticity and autocorrelation-consistent covariance matrix estimator. Under the null hypothesis that $\rho = 0$, the PP $Z_t$ and $Z_\pi$ statistics have the same asymptotic distributions as the ADF t-statistic and normalized bias statistics. One advantage of the PP tests over the ADF tests is that the PP tests are robust to general forms of heteroscedasticity in the error term $Ut$. 
Figure 10: Dependent Variables Stationarity

<table>
<thead>
<tr>
<th>Log Pay Bill I(0)</th>
<th>Log Pay Bill I(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Log Pay Bill I(0) graph" /></td>
<td><img src="image" alt="Log Pay Bill I(1) graph" /></td>
</tr>
<tr>
<td><strong>Non-stationary</strong></td>
<td><strong>Stationary</strong> $p = 0.00$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Log Numbers I(0)</th>
<th>Log Numbers I(1)</th>
<th>Log Numbers I(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Log Numbers I(0) graph" /></td>
<td><img src="image" alt="Log Numbers I(1) graph" /></td>
<td><img src="image" alt="Log Numbers I(2) graph" /></td>
</tr>
<tr>
<td><strong>Non-stationary</strong> $p = 1$</td>
<td><strong>Non-stationary</strong> $p = 0.4$</td>
<td><strong>Stationary</strong> $p = 0.00$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Log Rates I(0)</th>
<th>Log Rates I(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Log Rates I(0) graph" /></td>
<td><img src="image" alt="Log Rates I(1) graph" /></td>
</tr>
<tr>
<td><strong>Non-stationary</strong> $p = 0.84$</td>
<td><strong>Stationary</strong> $p = 0.00$</td>
</tr>
</tbody>
</table>

Source: Author’s Calculations
Table 5: Unit Root Test Results: Integrated order of each variable.

<table>
<thead>
<tr>
<th>Data Variable</th>
<th>$I(0)$</th>
<th>$I(1)$</th>
<th>$I(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay Bill</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Numbers</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Rates</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Live Register</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation Rate (CPI)</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Cohort &lt; 14 years</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Cohort 15-65 years</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Cohort &gt; 65 years</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s Calculations

5.5 Interpretation of Unit Root Testing

First Order Differencing

As shown in Table 5 no variables are stationary at levels. This means that the majority of data, as expected exhibits deterministic trends. As such first order differencing was required to achieve stationarity for Pay Bill, Rates, Real GDP, Revenue and the Live Register.

Second Order Differencing

After performing lag selection criteria and unit root testing, the variables representing demographics and numbers are integrated of order $I(2)$. In other words, the data will still not appear to be stationary after first differencing. It is therefore necessary to difference the data a second time to obtain a stationary series. Differencing the data of second order will reduce the explanatory power of the data. As such forming a model that examines $I(1)$ on $I(2)$ variables would be of limited insight.

However the results show that many of the variables are integrated of the same order. This allows for a cointegration regression analysis to be conducted between the variables integrated of the same order.
6. Methodology

Analysts face multiple econometric approaches depending on the data under examination, these include ARMA / ARIMA model, variance specific modelling such as ARCH-GARCH models, multiple equation analysis via Vector Autoregressive (VAR) models, and analysis of cointegrated time series. The type of econometric technique chosen depends to a large extent on the data, but also the theoretical justification.

Shown below is a framework under which the time series data dictates the type of model used, and is followed in this analysis.

Figure 11: Shrestha’s Framework for Model Selection.

![Shrestha's Framework for Model Selection](image)

Source: (Min B Shrestha, 2018)

The most appropriate techniques to use in this analysis are:

1) **Simple Trend Models**: ARIMA and ETS model of numbers, to help inform no policy, no shock forecasts of public service numbers growth in the short run.

2) **Auto-Regressive Distributed Lag (ARDL)** model for an analysis of the aggregate pay bill and rates equations,

3) **Cointegration error correction model** for the long run forecasts of public service numbers growth in the long run, due to demographic change.
The chosen methodology was deemed most appropriate for the following reasons:

(i) **Integration Order of the Data:**

As outlined in the previous section, there are differences in order of integration of the variables used. In other words, some variables are stationary after first order differencing while others after second order differencing.

(ii) **Cointegration Relationship:**

Economic analysis suggests that there often is a long run relationship between macroeconomic variables. Two or more predictive variables in a time-series model are cointegrated when they share a common stochastic drift. This means that cointegrated variables means and covariance’s are not dependent on time, (Nkoro, 2016). As outlined above, simple regression of non-stationary data may lead to spurious regression. To overcome this problem of non-stationarity, econometric analysis of time series data has increasingly moved towards the issue of cointegration (Nkoro, 2016). If this is the case, and there is theoretical justification to assert that the trend is common due to a long-run relationship, the data are considered to be cointegrated. If the data are cointegrated it can be examined through the lens of both long run and short run relationships (Asteriou, 2006).

**Preconditions for Cointegration:**

Traditionally, all of the series must be integrated of the same order. Then for an equilibrium to exist between the variables, there is a requirement that there is a linear combination of $Y_t$ and $X_t$ that is a stationary variable, (Asteriou, 2006). However, in 1998, Pearsan and Shin introduced an autoregressive distributed lag modelling approach to cointegration analysis for a long run relationship, irrespective of whether the underlying variables are $I(0), I(1)$ or a combination of both, (Pesaran, 1998).

As such, given the orders of integration of the data in this analysis, in addition to the strong common deterministic trends, this approach to establishing the Long and Short Run relationships between the variables is deemed appropriate.

(iii) **Theoretical Basis for use of multiple Lags**

Sometimes, the impact of a predictor which is included in a regression model will not be simple and immediate. For example, decisions made which will affect the pay bill can be announced at any time of the calendar year, however, due to the incremental system of remuneration it can
take up to 3 years for the full budgetary cost of pay increases to be realised. In time-series models, it is important to consider not only the elasticity of the independent variable on has on the dependent variable, but when it has the effect. Is the effect immediate? Does it emerge slowly? Is there an initial effect that goes away after a few periods? In order to answer these questions, one must estimate the lag distribution relating DV to IV.

(iv) Autoregressive Process (p):

In time series evaluation, it is assumed that the current value of the dependent variable, is influenced by its previous levels and trend. The autoregressive model accounts for this assumption as it specifies that the output variable depends linearly on its own previous values and on a stochastic term, an imperfectly predictable term. To not account for this in a dynamic process would be to miss-specify the model.

6.1 Forecast averaging

Economic forecasters often have a variety of different models and forecasts of the same variable from which to choose. These models and forecasts may differ in the underlying assumptions, or may employ different information. Traditionally, the forecasting decision was to choose a single forecasting method out of several competing alternatives.

However, forecasts from a given method may provide some useful information that is not conveyed in forecasts from other methods. Thus, instead of choosing a single forecasting method, a technique called forecast averaging considers information generated by several forecasts and then combines this information.

A number of studies have shown that averaging forecasts of multiple methods is more accurate than limiting a forecast to a single method. (Timmermann, 2006) offers a good overview of such studies, citing (Clemen, 1989) and (Makridakis, 2000) in particular.

The approach taken in this analysis is to employ a number of forecast techniques ranging from simple trend forecasting method to more complex autoregressive modelling, and then to take an average forecast of the multiple scenarios.

6.2 Reasons for not choosing other models

When choosing the methods to conduct this research, multiple of alternative methods were considered before choosing which one to employ. Evaluating the alternatives can avoid a single minded approach and reduces predetermined biases on how best to conduct the quantitative analysis.
ARCH / GARCH Model

ARCH) model is a statistical model for time series data that describes the variance of the current error term or innovation as a function of the actual sizes of the previous time periods' error terms. ARCH models are commonly employed in modelling financial time series that exhibit time-varying volatility and volatility clustering.

Vector Autoregressive Models (VAR)

A Vector Autoregressive (VAR) model is a stochastic process model used to capture the linear interdependencies among multiple time series. They are very popular among time series of macroeconomic data. The usefully model the two-way casual relationships between variables, for example, when GDP increases often so does consumption, but when consumption rises so does GDP, so a vector od DV is created. Therefore, in some respect, VAR models could be considered atheoretical (Asteriou, 2006).

In this analysis, the model is founded in the literature and the theory on the topic. So from the literature, an exogenous relationship between the DV and the IV’s is postulated. As such modelling for endogeneity is not appropriate in this instance.
7. Model Specification

7.1 Model 1: Numbers Forecasts: ARIMA & ETS Forecast Average Model.

As stated in the objectives of this paper, a core aim of this paper is to improve the forecasting of numbers growth over the short term. As such, it seems an appropriate place to start for models using level and trend data to inform short-run expectations.

Error, Trend, Seasonal (ETS) Modelling

ETS forecasts are predictions made to the level and trend of a time series via a weighted sum of past observations, but the model explicitly uses an exponentially decreasing weight for past observations, (Athanasopoulos, 2018).

Exponential smoothing was proposed in the late 1950s (Brown, 1959); (Holt, 1957); (Winters P. R., 1960). Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older (Athanasopoulos, 2018). In other words, the more recent the observation the higher the associated weight.

(Holt, 1957)) extended simple exponential smoothing to allow the forecasting of data with a trend. This method involves a forecast equation and two smoothing equations (one for the level and one for the trend):

Forecast equation  \[^y_t + h|t = \ell_t + h\beta t\]

Level Equation  \[\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + h\beta (t-1))\]

Trend Equation  \[b_t = \beta^* (\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}\]

The forecasts generated by Holt’s linear method display a constant trend (increasing or decreasing) indefinitely into the future. Empirical evidence indicates that these methods tend to over-forecast, especially for longer forecast horizons. Motivated by this observation, (Gardner, 1985) introduced a parameter that “dampens” the trend to a flat line sometime in the future. In conjunction with the smoothing parameters \(\alpha\) and \(\beta^*\) (with values between 0 and 1 as in Holt’s method), this method also includes a damping parameter \(0 < \varphi < 1\).

Auto-Regressive Moving Average (ARIMA) Modelling

ARIMA models provide another approach to time series forecasting. Exponential smoothing and ARIMA models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem. While exponential smoothing models are based on a
description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

In an auto-regression model, I forecast the variable of interest using a linear combination of past values of the variable. The term auto-regression indicates that it is a regression of the variable against itself.

7.2 Model 2: Numbers Demography Cointegration Model

\[ \text{Numbers} = \alpha + \text{Numbers}_{t-1} + \beta_{i,n} \text{Cohort < 14 years old}_{t-n} \]
\[ + \beta_{i,n} \text{Cohort 15 – 65 years old}_{t-n} + \beta_{i,n} \text{Cohort > 65 years old}_{t-n} \]

Based on the literature review, trends in demographic change place additional pressure on public services. As such a number of variables to model the demographic pressure effect are included. These are:

1. Total Population
2. Cohort less than 14 years old
3. Working Age Population
4. Cohort over age 65 years old.

The economic context and public finances are considered key determinants of public service employment according to the literature. However, given that the variable numbers is integrated of \(I(2)\), it is limited to being analysed with data of equal integration and where a cointegration relationship exists. This limits this model to examining the causal relationship between Numbers and Demography, while excluding the macroeconomic and public finance considerations identified in the literature. Using the Johansen test for integration, we find there the \(I(2)\) variables do share a long run deterministic trend.

7.3 Model 3: ARDL FMOLS Public Pay Bill and the Economic Cycle:

\[ \text{Paybill} = \alpha + \text{Paybill}_{t-1} + \beta_{i,n} \text{GDP}_{t-n} + \beta_{i,n} \text{Revenue}_{t-n} + \beta_{i,n} \text{CPI}_{t-n} + \beta_{i,n} \text{Live Reg}_{t-n} \]
\[ + \text{Election Dummy} + \text{Recession Dummy} \]

The purpose of this model if to examine at an aggregate level, how the pay bill relates to various independent variables identified in the literature. As discussed in section 5, due to the order of integration of some of the time series data, numbers and demographic variables cannot be included, but are examined in a separate model.
7.4 Model 4: ARDL FMOLS Public Service Rates and the Economic Cycle

\[ Rates = \alpha + Rates_{t-1} + \beta_{1,n}GDP_{t-n} + \beta_{2,n}Revenue_{t-n} + \beta_{3,n}CPI_{t-n} + \beta_{4,n}Live Reg_{t-n} + Election Dummy \]

Identified in the literature review, as the key determinants of public sector wages are, economic growth, public finances, general prices, house process, wage inflation in the wider economy, and the cycle of the labour market. As such, where possible these variables have been included to reflect the domestic and international literature in, in the Rates specific equation. As discussed in the data section, the Rates variable is the unit labour cost of public service employees, deflated by inflation.

From the outset, there are variables which would have been include in this model, based on the literature review, had the data been available. Unfortunately, some data series where not of an appropriate standard to incorporate into this analysis, as outlined below.

Rates Equation Limitations:

(i) House Prices

A consistent time series of data was not available to model House Prices or Rents into this equation. Given the impact of housing/rental on overall cost of living identified in 4.3, this is a limitation.

There is an archived time series of House Price data from Department of Housing hosted at the CSO, broken down by new and second hand house prices. It should be noted that these are prices are not mix adjusted and only cover dwellings bought with a mortgage. They are, however, the only official house price statistics for the period. Due to these issues and the lack of comparability to the current national house price series, it was decided to omit this variable from the analysis with the aim of continuing work with the CSO to find a solution.

(ii) Comparable Wages:

As outlined in the literature an obvious omitted variable from the Rates equation is an indicator of a comparable wage. The CSO’s Earnings, Hours and Employment Cost Survey (EHECS) has good quarterly data, broken down by NACE economic sector.

However, this series only dates back to 2008. The CSO do have a longer times series of an average industrial weekly wage, which was considered for this analysis, however, the CSO have published an econometric analysis of the public/private sector pay differential, (CSO, 2014), which describes why

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5 https://www.cso.ie/px/pxiirestat/Statire/SelectVarVal/Define.asp?maintable=HSA06&PLanguage=0
this would not be a comparable metric. The average public sector earnings are higher than private sector earnings for a variety of reasons including: public sector employees are on average older than private sector employees, are more likely to have completed third level education and are more likely to be a member of a trade union. Unable to control for these differences, and in the absence of a better alternative, this remains an omission from the analysis.
8. Results

8.1 Numbers Results – Model 1: ARIMA, ETS Numbers and Forecast Average

Table 6: No Policy Change Short Run Numbers Forecast

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Average Forecast</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2019 (f)</td>
<td>347,149</td>
<td>339,359</td>
<td>342,032</td>
<td>3.5%</td>
</tr>
<tr>
<td>2020 (f)</td>
<td>355,986</td>
<td>345,715</td>
<td>349,156</td>
<td>2.21%</td>
</tr>
</tbody>
</table>

Source: Authors own calculations

ARIMA and ETS models are useful as a short run trend forecast. However, notice the large range between the min and max values. The regression model of Numbers and Demography provide the upper higher estimates for this forecast average, while the ARIMA and ETS resulted in the lower estimate. The Short Run forecast figure is therefore the average of the trend models and the long run model, and we consider this a conservative estimate in the Short Run.

Figure 12: Simple Forecast Models of Numbers: ARIMA and ETS Forecasts

Source: DPER administrative Data and Authors own calculations

As outlined in section 3.2, numbers have grown between Budget and Outturn in response to policy decisions, making the pay bill difficult to forecast as a consequence. Using an average forecast of the ARIMA, ETS and regression model, numbers by end 2020 are forecast to increase to c.350,000.
8.2 Numbers Results – Model 2: Numbers Demography Cointegration Model

Presented below are the results of Model 2, outlined in section 7.2. The correlation coefficients are present in natural logarithmic terms, where the coefficient can be interpreted as a relationship in elasticity terms.

Table 7: Numbers and Demographic Regression Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohort &gt; 65 Years Old</td>
<td>1.5845</td>
<td>0.3322</td>
<td>4.6554</td>
<td>0.0000</td>
</tr>
<tr>
<td>Cohort between 15 and 65 years old</td>
<td>3.0583</td>
<td>0.2007</td>
<td>14.9644</td>
<td>0.0000</td>
</tr>
<tr>
<td>Cohort &lt; 14 years old</td>
<td>-0.9379</td>
<td>0.2408</td>
<td>-3.8349</td>
<td>0.0001</td>
</tr>
<tr>
<td>Recession Dummy Variable (Moratorium years)</td>
<td>-0.0198</td>
<td>0.0174</td>
<td>-1.1396</td>
<td>0.2563</td>
</tr>
<tr>
<td>C</td>
<td>-46.0188</td>
<td>5.3858</td>
<td>-8.5444</td>
<td>0.0000</td>
</tr>
<tr>
<td>Trend</td>
<td>-0.0130</td>
<td>0.0020</td>
<td>-6.6362</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Source: Authors own calculations, using Eviews software.

8.3 Numbers Results – Summary, Forecast and Policy Implications:

- This analysis was able to examine the effect of demography on public service numbers.
- Overall, there is a strong positive correlation between population increase and growth in public service numbers to meet the delivery of public services.
- In examining the compositional effect of population increases, a 1% increase in the age cohort between 15 and 65 years old, will place an additional 3% increase on public service numbers over the long run.
- A 1% increase in the Old Age Population, will place an additional 1.6% increase in public service numbers over the long run. Given the relative size of this cohort as a percentage of overall population.
- Over the Long-run, there is shown to be a marginal impact of the less than 14 years old cohort on public service numbers. This is intuitive given the size of this cohort relative to other cohorts.
Given that Numbers have found to be positively correlated with population growth and compositional change, CSO population estimates are used to construct a no policy change forecast of how numbers may respond to projected increases in each population cohort examined in this model.

Figure 13 below shows the forecasted population growth according to CSO M2F2 scenario projections. Under these projections, overall population growth of c.10% is expected, with the 65+ cohort to increase by 30% by 2025. The cohort aged less than 14 years old is forecast to decrease by c. 4% between 2016 and 2025.

Figure 13: Central Statistics Office M2F2 2016 Forecasts for certain Cohorts:

Source: Authors own calculations of CSO M2F2 population projections.

Given the long run coefficients from the Numbers and Demography model, coupled with the expected increases in population cohorts over the medium term, pressure to expand public service employment is likely over the medium term. The forecast below are an attempt to quantify the magnitude of that pressure based on current service levels.

Pre-crisis numbers peaked at 325,117 before declining during the crisis to 292,215 through the implementation of a recruitment moratorium. This reduction proved to be temporary as numbers rebounded post crisis: Q2 2019 data shows public service employment is c.10,000 above pre crisis levels and increasing.
Using the CSO’s M2F2 population projections detailed above, this paper forecasts that public service employment could, in the absence of a policy change of abatement or reduction in public services per capita, exceed 410,000 by 2025 (24% increase on 2018 figures, an average of 3.3% per annum).

Of course, the further one forecasts, the greater degree of uncertainty pertains. Nevertheless the benefit of this analysis from a policy making perspective, is to establish a public service employment trajectory, if public service numbers were to match increases in population on a like for like basis of existing level of service. Put another way this forecasts what public service employment could be if the current service mix were to remain constant for future cohorts of the population.

**Figure 14: No policy change, no shock, Numbers and Demography related forecast.**

*Source: Authors own calculations, using Eviews software.*
8.4 Aggregate Pay Bill – ARDL FMOLS Model of Public Service Rates and Economic Cycle

Presented below are the results of Model 3, outlined in section 7.3. The correlation coefficients are present in natural logarithmic terms, where the coefficient can be interpreted as a relationship in elasticity terms.

Table 8: Aggregate Pay Bill Regression Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Election</td>
<td>-0.3554</td>
<td>0.16801</td>
<td>-2.1157</td>
<td>0.0364</td>
</tr>
<tr>
<td>CPI</td>
<td>2.1750</td>
<td>1.0392</td>
<td>2.0929</td>
<td>0.0385</td>
</tr>
<tr>
<td>GDP</td>
<td>0.9256</td>
<td>0.4837</td>
<td>1.9135</td>
<td>0.0581</td>
</tr>
<tr>
<td>Recession Dummy</td>
<td>-0.3558</td>
<td>0.2385</td>
<td>-1.4918</td>
<td>0.1384</td>
</tr>
<tr>
<td>Revenue</td>
<td>-0.6261</td>
<td>0.5366</td>
<td>-1.1670</td>
<td>0.2455</td>
</tr>
<tr>
<td>Live Register</td>
<td>-0.2459</td>
<td>0.2485</td>
<td>-0.9897</td>
<td>0.3243</td>
</tr>
</tbody>
</table>

Source: Authors own calculations, using Eviews software.

8.5 Pay Bill Results – Summary, Forecast and Policy Implications

- At an aggregate level, the pay bill is positively correlated with the economic cycle.
- The pay bill is most strongly positively correlated with prices at the 5% confidence interval.
- A 1% change in inflation, has corresponded to a 2% change in the pay bill over the long run.
- Similarly, there is a positive and statistically significant relationship between the pay bill and economic output, at the 10% confidence interval.
- A 1% change in economic growth has corresponded to a 1% change in the pay bill over the long run.
- There is shown to be a negative relationship between the pay bill and other variables including recession, liver register and total revenue, beyond the 90% confidence interval. For both recession and the live register this is in line with the literature, however this is counterintuitive for revenue. Although attempted to be controlled for in this model, it may be the experience of 2008.7

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7 PBO publication found here, “2008-2009: Tax revenues fall significantly but public pay expenditure continues to grow”
No Policy Change, No Shock, Pay Bill Forecast

Based on Department of Finance macroeconomic forecasts, published in the April 2019 Stability Programme Update, Figure 15 shows how the Exchequer pay bill may respond to movements in the wider economy.

In 2018 the pay bill increased by 7.3% to c. €18bn. Under a no policy change scenario, this paper forecasts the pay bill will grow by c. 8% per annum, increasing to €24bn by 2022.

Figure 15: No Policy Change, No Shock Aggregate Pay Bill Forecast

Source: Authors own calculations, using Eviews software.

As the analysis of the pay bill is based on the long run relationship between the pay bill and the economic cycle, and the pay bill has historically grown in a pro-cyclical manner, the forecasts can be understood as a continuation of a pro-cyclical pay policy.
8.6 ARDL FMOLS Model Public Service Rates and Economic Cycle:

Presented below are the results of Model 4, outlined in section 7.4. The correlation coefficients are present in natural logarithmic terms, where the coefficient can be interpreted as a relationship in elasticity terms.

Table 9: Rates and Economic Cycle Regression Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>1.983258</td>
<td>0.972179</td>
<td>2.040014</td>
<td>0.0433</td>
</tr>
<tr>
<td>GDP</td>
<td>1.428162</td>
<td>1.033623</td>
<td>1.381705</td>
<td>0.1694</td>
</tr>
<tr>
<td>Revenue</td>
<td>-0.493106</td>
<td>0.743271</td>
<td>-0.663427</td>
<td>0.5082</td>
</tr>
<tr>
<td>Election</td>
<td>-0.052344</td>
<td>0.093633</td>
<td>-0.559037</td>
<td>0.5771</td>
</tr>
<tr>
<td>Live Register</td>
<td>0.128604</td>
<td>0.296646</td>
<td>0.433526</td>
<td>0.6653</td>
</tr>
</tbody>
</table>

Source: Authors own calculations, using Eviews software.

8.7 Rates Results Summary

- In line with the literature, rates are shown here to have a strong positive correlation with Prices and the Macroeconomy. However, the results show limited statistical significance.

- Given the limitation of the model, notably the absence of sufficient comparable wages, house prices or rental data, further analysis may be required to improve this model. Although inconclusive, this work has at least pointed to the specific data gaps researchers can focus on to do this.
9. Policy Implications and Next Steps

- This work clarifies how the Exchequer pay bill and public service numbers may be expected to evolve over time, based on current public service provision and forecasted changes in the population structure and the economy. As such it has broad policy implications.

- As the analysis of the pay bill is based on the long run relationship between the pay bill and the economic cycle, and the pay bill has historically grown in a pro-cyclical manner, the forecasts can be understood as a continuation of a pro-cyclical pay policy.

- The scale and composition of demographic change will put pressure on public service numbers, regardless of the economic cycle.

- Importantly the forecasts of continued growth in public service number and the pay expenditure indicates a level of expansion that may have to be accommodated in Current Expenditure Ceilings under a no policy change scenario.

- To improve the ability to model Public Service rates and the economic cycle, DPER will engage with the CSO to establish whether a long run and consistent time series of comparable wages and housing prices can be constructed.

- This work will inform expenditure planning under alternative scenarios including various potential Brexit outcomes.
Quality Assurance Process

To ensure accuracy and methodological rigour, the author engaged in the following quality assurance process.

✔ Internal/Departmental
  ✔ Line management
  ✔ Spending Review Sub-group and Steering group
  ✔ Other divisions/sections – Central Votes Section and the Public Service Reform and Delivery Office.
  ✔ Peer review (IGEES network, seminars, conferences etc.)

✔ External
  ☐ Other Government Department
  ☐ Advisory group
  ☐ Quality Assurance Group (QAG)
  ☐ Peer review (IGEES network, seminars, conferences etc.)
  ☐ External expert

✔ Other (relevant details)

  The econometric analysis contained in this paper was conducted by Evan Walker under supervision of Dr Micheál Collins, University College Dublin, as part of a Master’s in Public Policy dissertation.
Bibliography


